CWTS
Meaningful metrics

Dynamical models of mutually antagonistic groups

Laboratoire Informatique d'Avignon

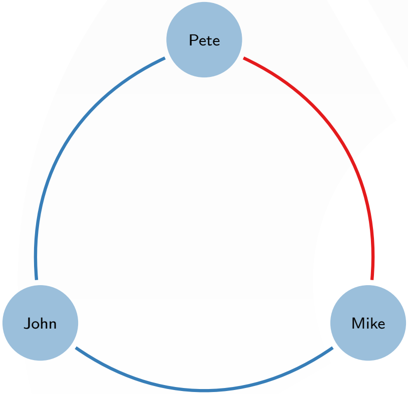
18 March 2016

V.A. Traag

Universiteit Leiden

Fighting among friends

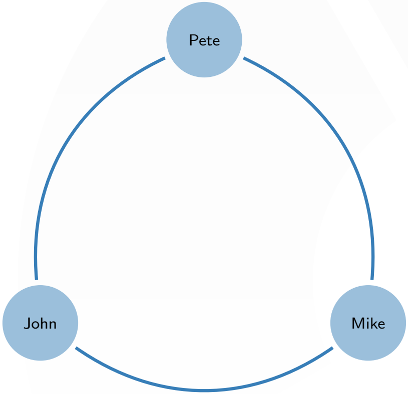
Two friends are fighting. (unstable)



CWTS
Meaningful metrics

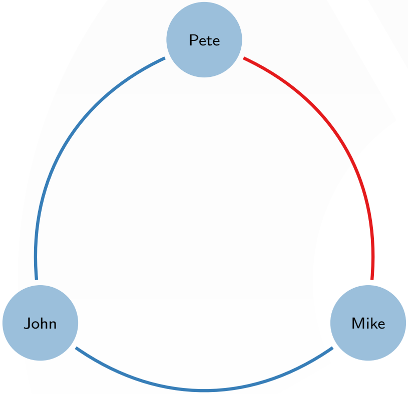
Fighting among friends

All friends, everybody happy. (stable)



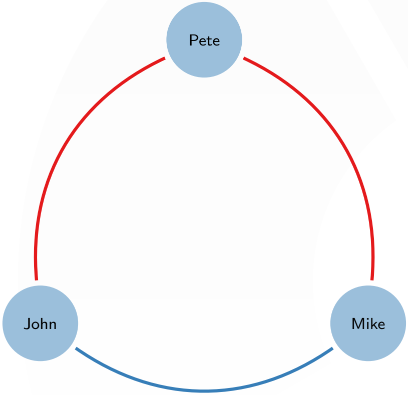
Fighting among friends

Two friends are fighting. (unstable)



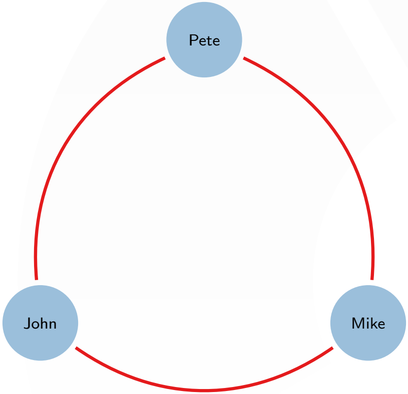
Fighting among friends

My enemy's enemy is my friend. (stable)



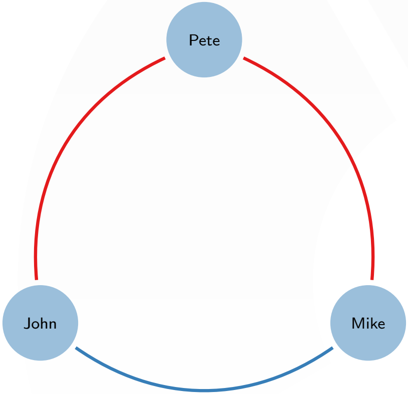
Fighting among friends

Mutual enemies. (unstable)

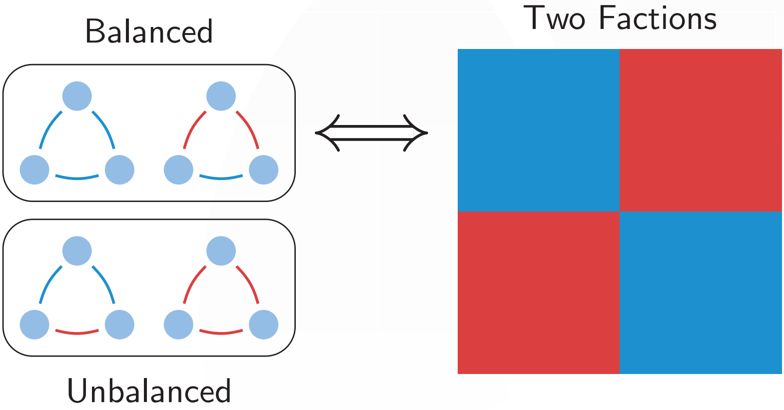


Fighting among friends

My enemy's enemy is my friend. (stable)

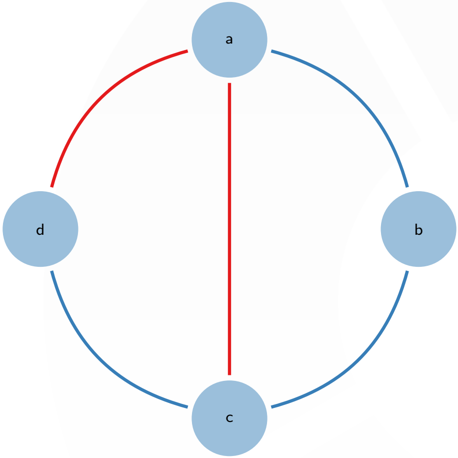


Social balance



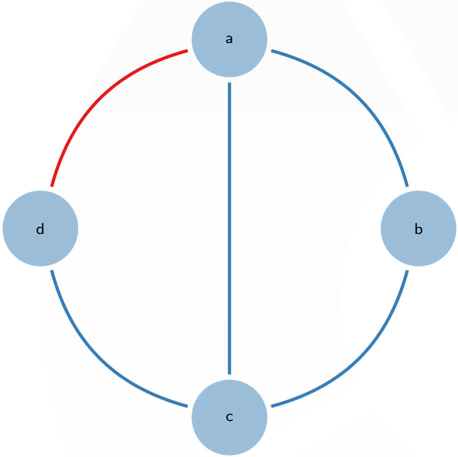
Discrete social balance dynamics

One unbalanced triad, one balanced triad.

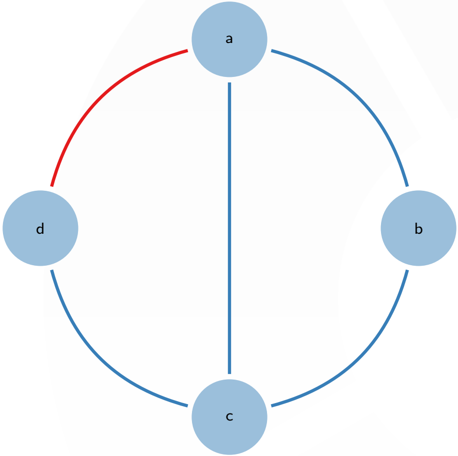


Discrete social balance dynamics

Flip sign of a link.

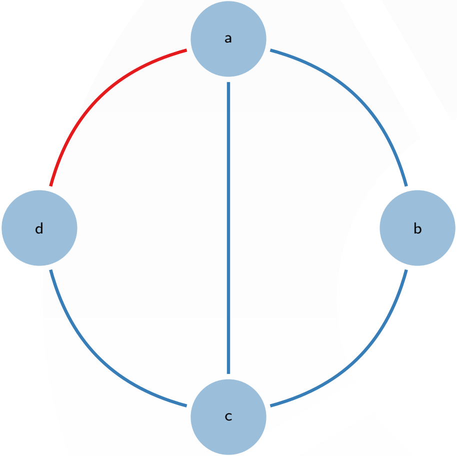


Only "improving" moves can get stuck (no flip improves).



Discrete social balance dynamics

Discrete dynamics can take a long time to converge.



Continuous social balance dynamics

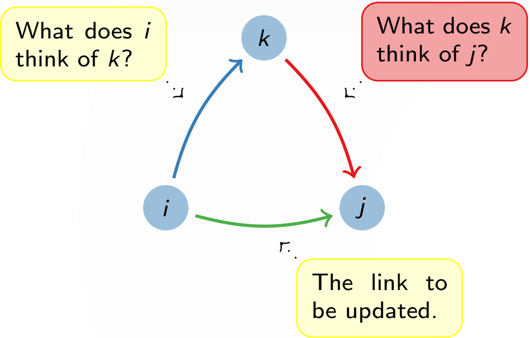
Continuous (time) approach

- No more discrete positive or negative links.
- Indicate x_{ij} the opinion i has of j.
- Social balance: $x_{ij}x_{jk}x_{ki} > 0$ for all i, j, k.
- Dynamics $\dot{x}_{ij}$ such that $\lim_t X(t)$ socially balanced?

CWTS Meaningful metrics

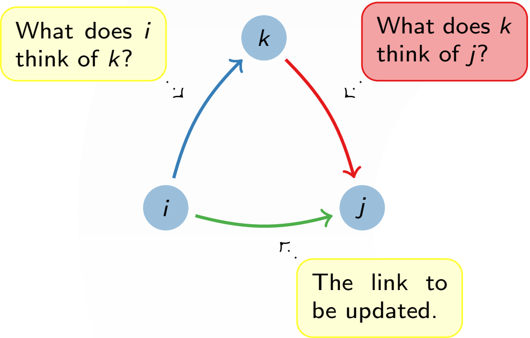
Social balance, gossiping dynamics

$$\dot{x}_{ij} = \sum_k x_{ik} x_{kj}$$



Social balance, gossiping dynamics

$$\dot{X} = X^2$$



Dynamics, symmetric case

- Symmetric initial condition easy
- Decompose $X = U \Lambda U^T$
- Then

$$\dot{X} = X^2$$
$$U^T \dot{X} U = U^T X U U^T X U$$
$$\dot{\Lambda} = \Lambda^2$$

- Solution simple $\lambda_i(t) = \frac{\lambda_i(0)}{1-\lambda_i(0)t}$.
- Blows up at time $t = \frac{1}{\lambda_i(0)}$ for $\lambda_i(0) > 0$.
- If $\lambda_1 > \lambda_2$ and $\lambda_1 > 0$ then normalised converges to

$$\frac{X(t)}{|X(t)|} \sim u_1 u_1^T \sim \begin{pmatrix} + & - \\ - & + \end{pmatrix}.$$

- Converges to social balance.

CWTS Meaningful metrics

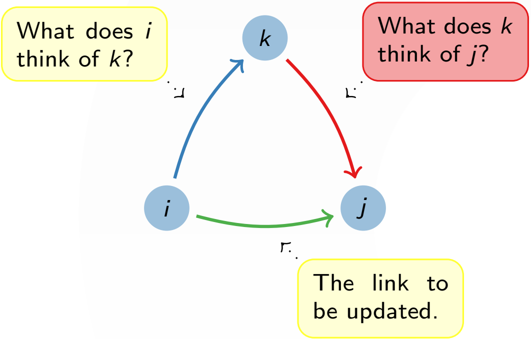
- Non-symmetric (general) initial conditions more difficult.
- For $\dot{X} = X^2$, use Jordan Normal Form. Solutions blow up to

$$\frac{X(t)}{|X(t)|} \sim u_1 v_1^T \sim \left(\begin{array}{cc|cc} + & - & + & - \\ + & - & + & - \\ \hline - & + & - & + \\ - & + & - & + \end{array}\right).$$

- Does **not** converge to social balance.

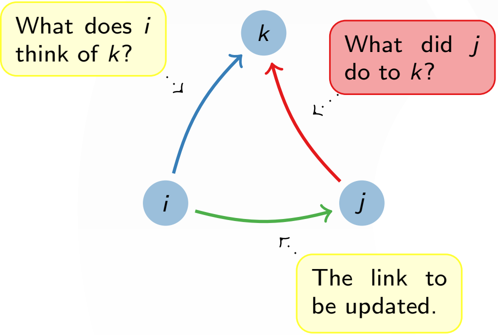
CWTS Meaningful metrics

$$\dot{x}_{ij} = \sum_k x_{ik} x_{kj}$$

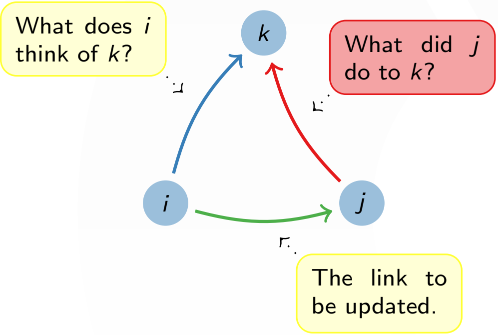


CWTS Meaningful metrics

$$\dot{x}_{ij} = \sum_k x_{ik} x_{jk}$$



$$\dot{X} = XX^T$$


What does i think of k?
k
What did j do to k?
i
j
The link to be updated.

Dynamics, general case

- Symmetric case, identical behaviour.
- To solve general case, decompose

$$S = \frac{X + X^T}{2}, A = \frac{X - X^T}{2}.$$

leading to

$$\dot{S} = (S + A)(S - A), \dot{A} = 0.$$

- Change of variables $Z = V^T e^{tA_0} S e^{-tA_0} V$, and with D^2 the diagonal of $-A^2$, leads to

$$\dot{Z} = Z^2 + D^2,$$

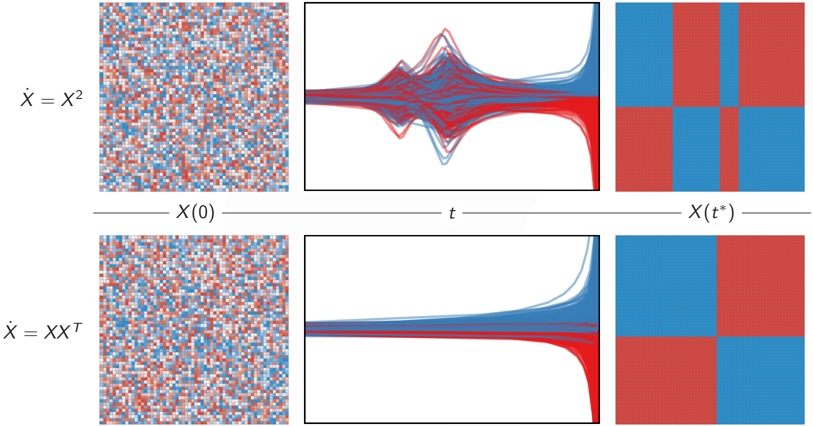
- Solutions blow up to

$$\frac{X(t)}{|X(t)|} \sim ww^T \sim \begin{pmatrix} + & - \\ - & + \end{pmatrix}$$

for some w.
- Almost every initial condition converges to social balance.

CWTS Meaningful metrics

Solutions



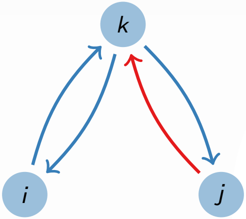
Traag, Van Dooren & De Leenheer. *PLoS ONE* **8**, e60063 (2013).

Motivation, evolutionary game theory

Agents either
Cooperate or
Defect

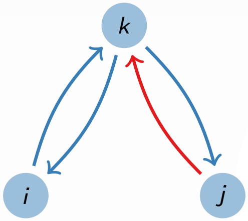
$$\begin{array}{c|cc} & C & D \\ \hline C & b-c & -c \\ D & b & 0 \end{array}$$

i and k play: i and k cooperate.

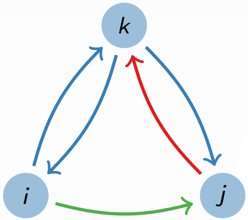


CWTS
Meaningful metrics

j and k play: k cooperates, j defects.



Should i cooperate or defect with j?



Reputation update

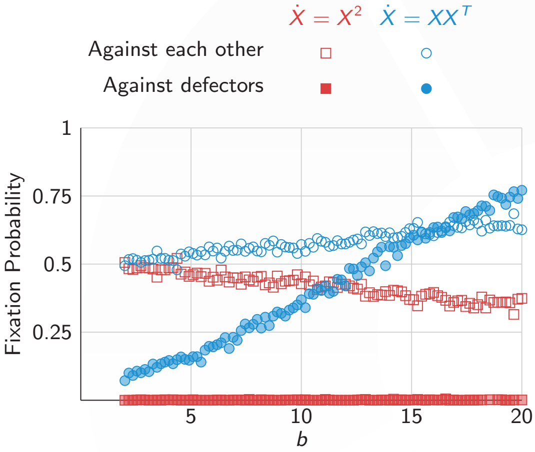
Did j cooperate or defect with k?

Reputation of k in eyes of i?

$$\begin{array}{c|cc} & C & D \\ \hline G & + & - \\ B & - & + \end{array}$$

Effect on x_{ij}

Cooperation



Traag, Van Dooren & De Leenheer. *PLoS ONE* **8**, e60063 (2013).

Conclusions

Conclusions

- Discrete dynamics may not lead to social balance.
- $\dot{X} = X^2$ lead to social balance only in symmetric case.
- $\dot{X} = XX^T$ (almost) always leads to social balance.
- $\dot{X} = XX^T$ leads to cooperation, but also to conflict.

Open questions

- Generalized social balance (> 2 groups).
- Dynamics on sparse networks.
- Empirical corroboration of model.

CWTS
Meaningful metrics

Thank you!

Questions?

Email v.a.traag@cwts.leidenuniv.nl Twitter @vtraag