Algebraic Graph-theoretic Measures of Conflict

Jérôme Kunegis

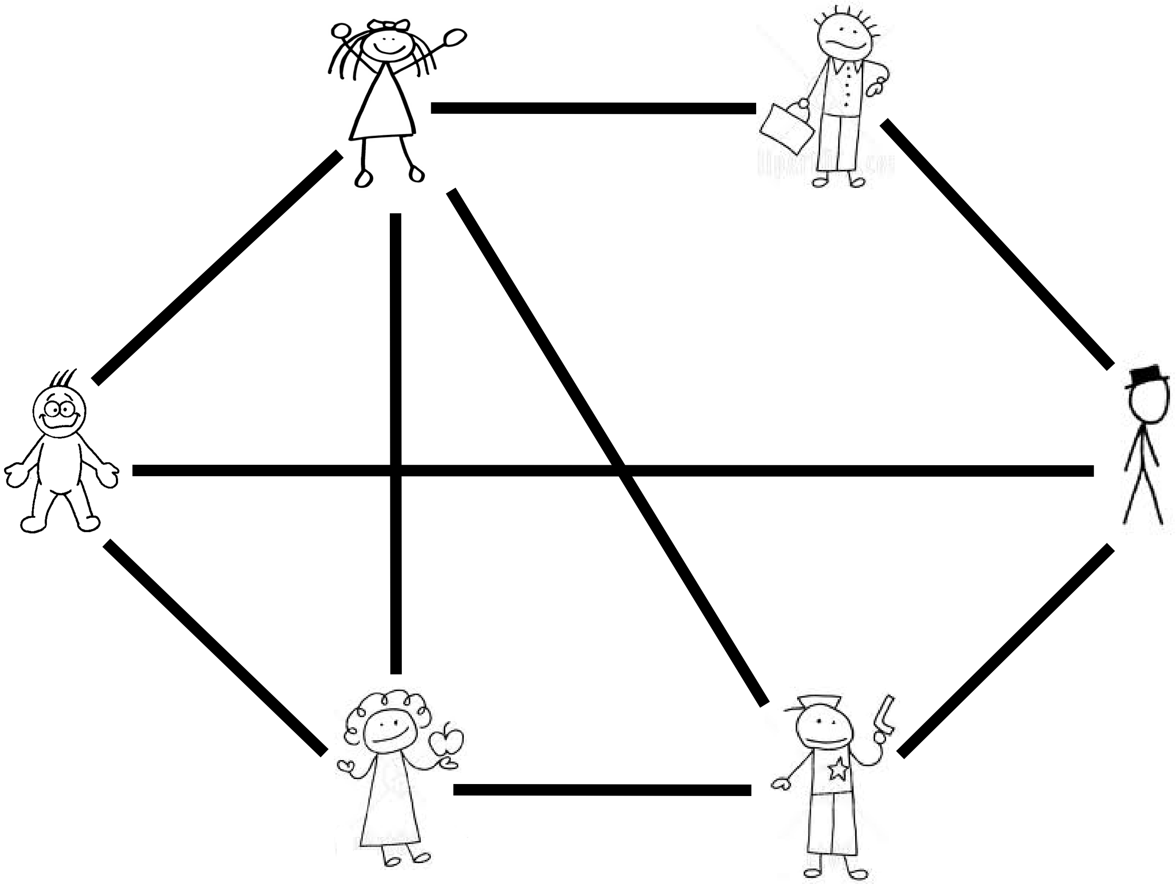
Based on work performed in collaboration with Christian Bauckhage, Andreas Lommatzsch, Stephan Spiegel, Jürgen Lerner, Fariba Karimi and Christoph Carl Kling

Journée Graphes et Systèmes Sociaux (JGSS), March 18, 2016, Avignon

People and Knowledge Networks

Institute for Web Science and Technologies · University of Koblenz-Landau, Germany

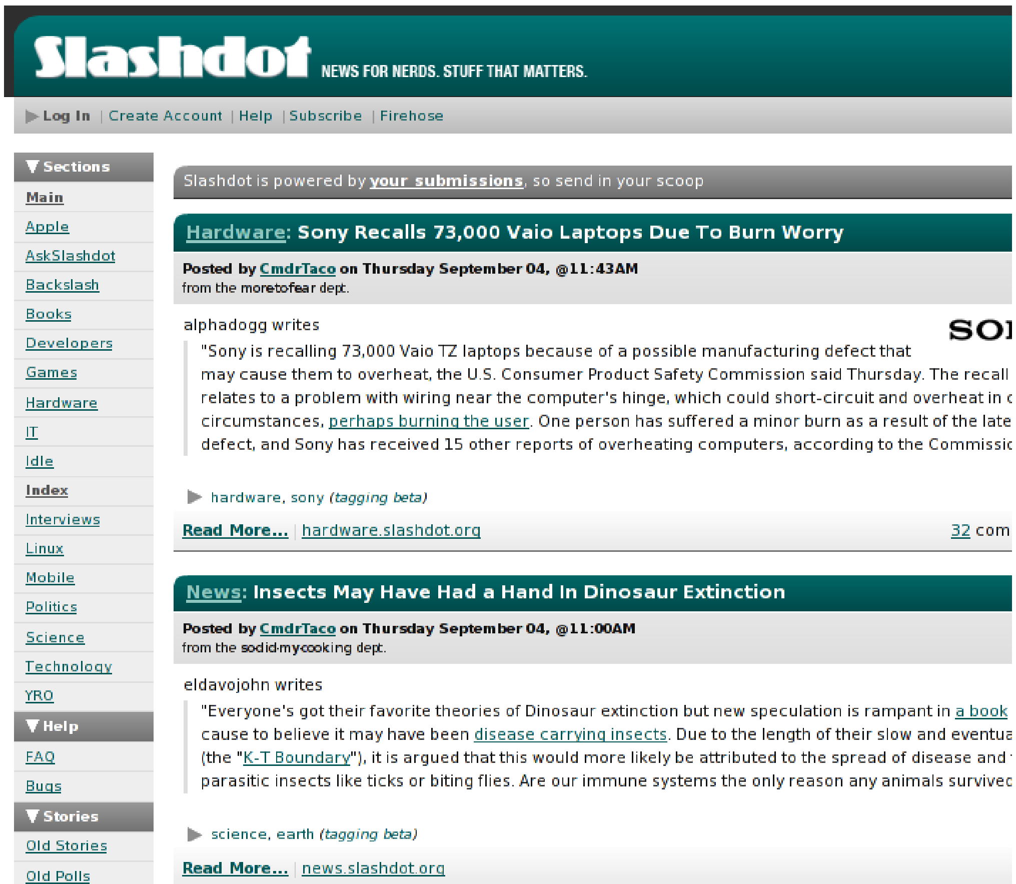
Social Network

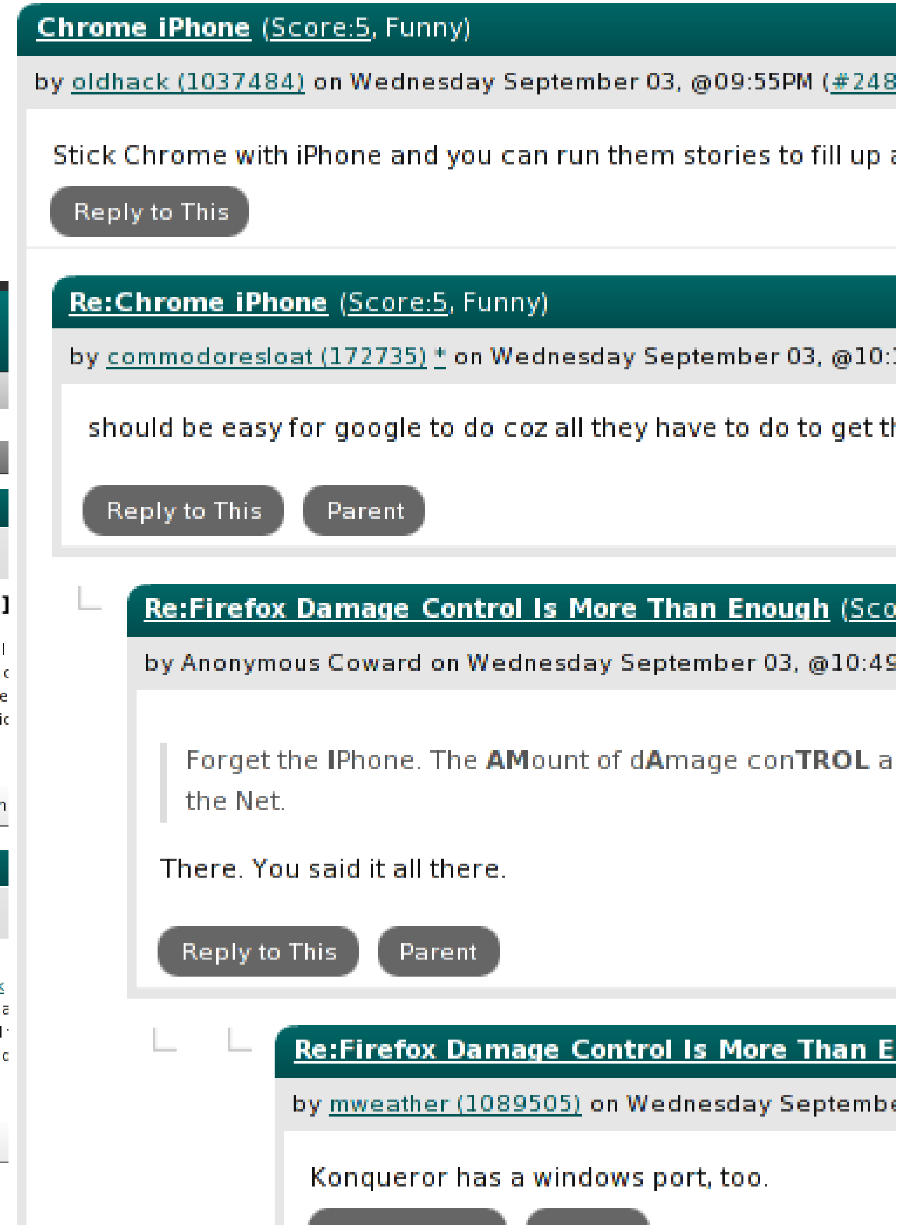


Slashdot

News for Nerds. Stuff that matters.

http://slashdot.org/

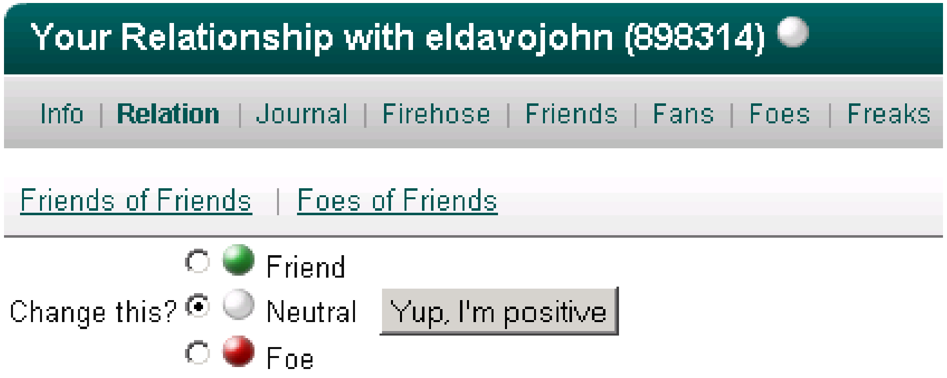




The Slashdot Zoo

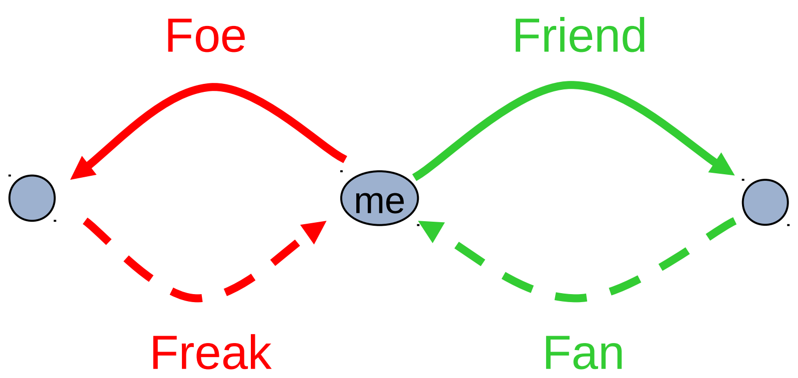
Slashdot Zoo: Tag users as ***friends*** and ***foes***

Graph has ***two*** types of edges: friendship and enmity



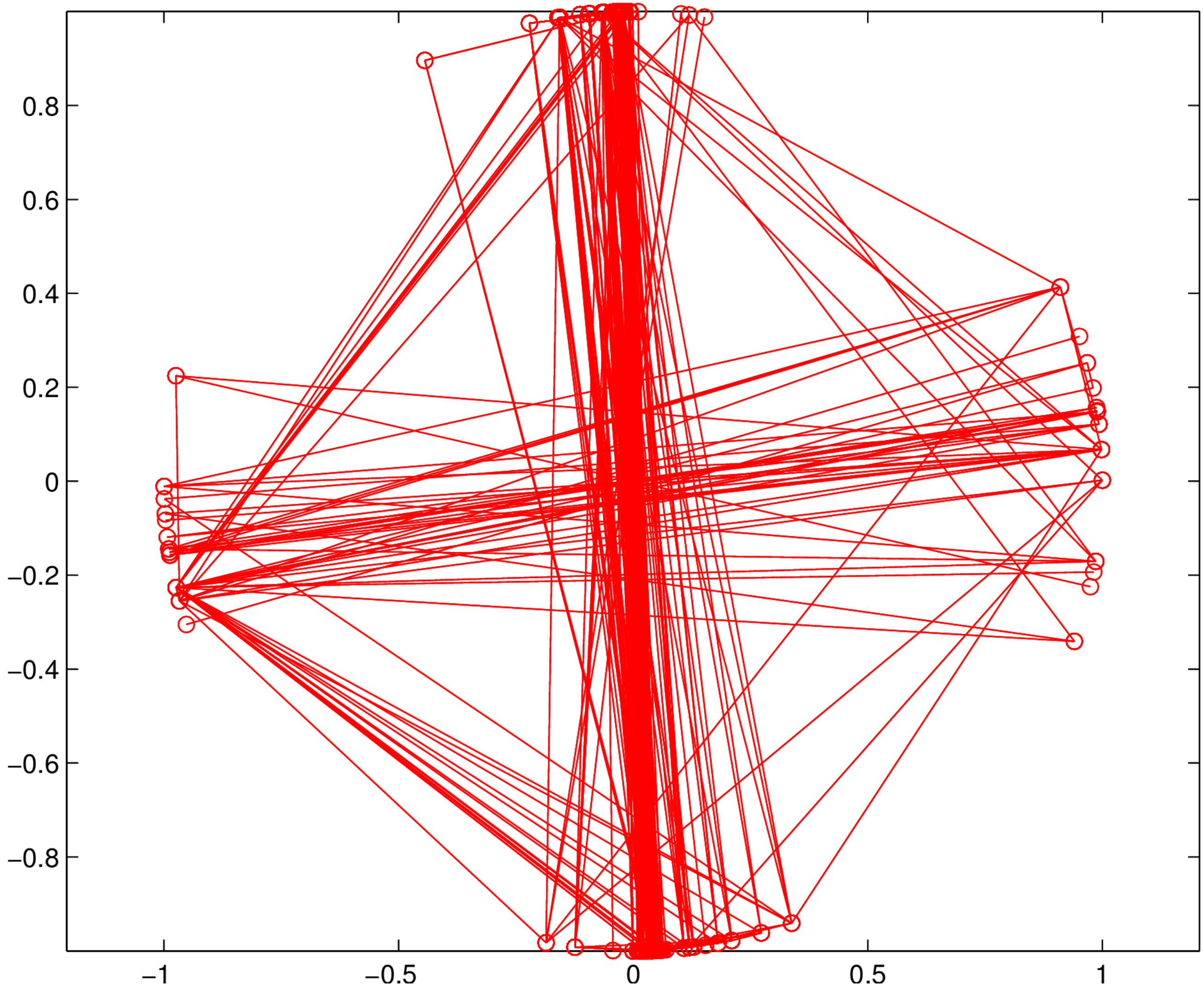
Signed Directed Social Network

You are the fan of your friends and the freak of your foes.

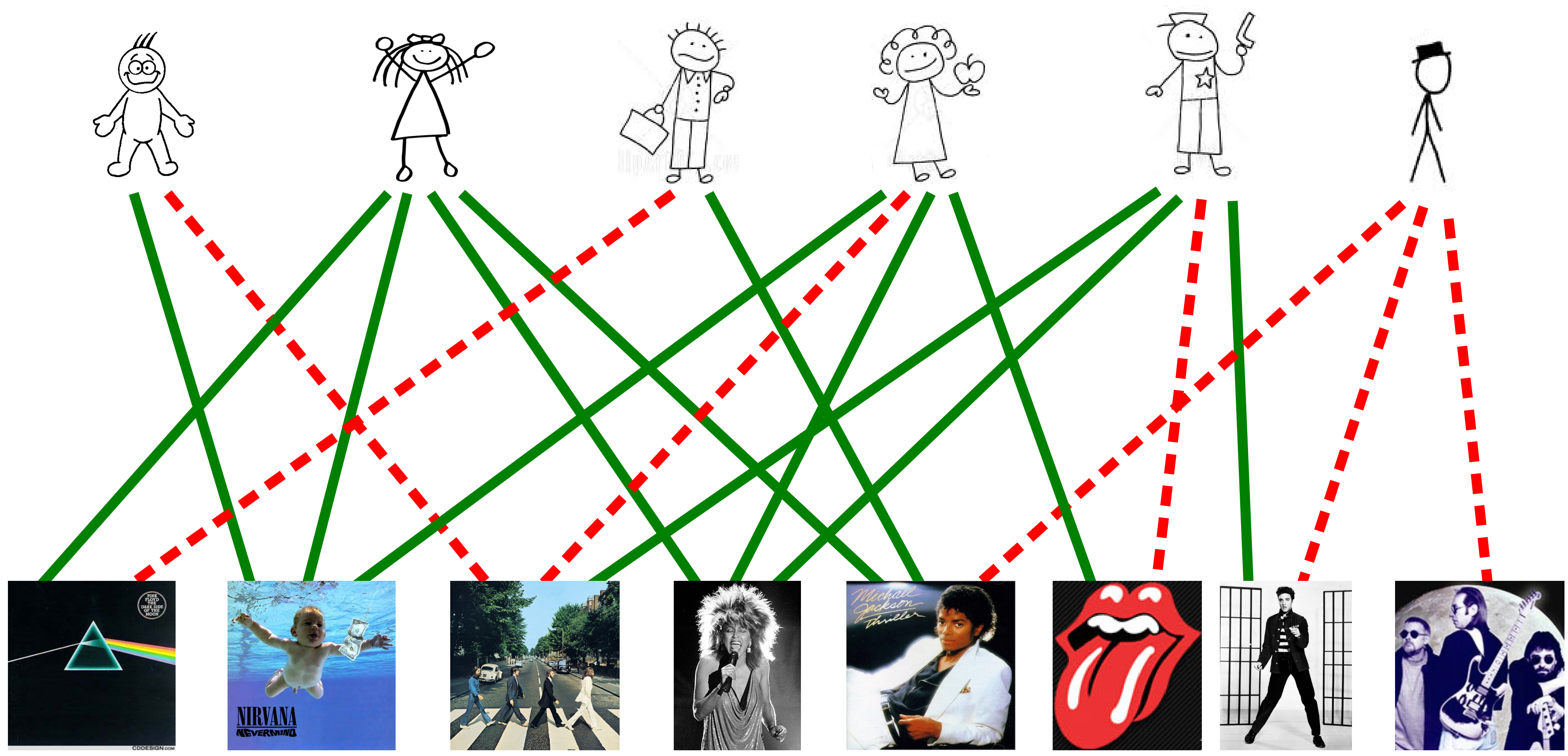


The resulting graph is sparse, square, asymmetric and has signed edge weights.

Wikipedia Edit Wars



Signed Bipartite Networks



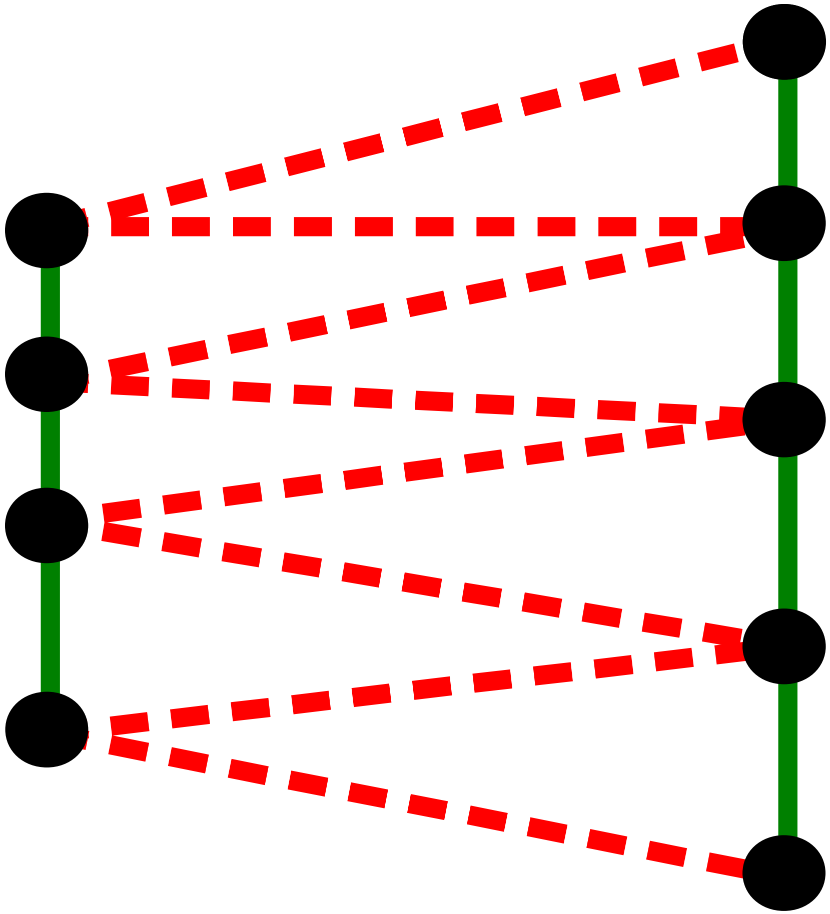
Other Signed Networks

Mv	Dutch college	HumanSocial	D	±	32	3,062
EP	Epinions trust	Social	D	±	131,828	841,372
HT	Highland tribes	HumanSocial	U	±	16	58
SZ	Slashdot Zoo	Social	D	±	79,120	515,397
CO	Wikipedia conflict	OnlineContact	U	±	118,100	2,917,785
EL	Wikipedia elections	OnlineContact	D	±	7,118	103,675
W2	WikiSigned	OnlineContact	D	±	138,592	740,397
AR	Amazon ratings	Rating	B	*	3,376,972	5,838,041
Bx	BookCrossing (ratings)	Rating	B	*	263,757	433,652
ER	Epinions product ratings	Rating	B	*	876,252	13,668,320
Fr	Filmtipset	Rating	B	*	144,671	19,554,219
J1	Jester 100	Rating	B	*	73,521	4,136,360
J2	Jester 150	Rating	B	*	50,832	1,728,847
LI	Libimseti.cz	Social	D	*	220,970	17,359,346
M1	MovieLens 100k	Rating	B	*	2,625	100,000
M3	MovieLens 10M	Rating	B	*	80,555	10,000,054
M2	MovieLens 1M	Rating	B	*	9,746	1,000,209
NX	Netflix	Rating	B	*	497,959	100,480,507
MO	Residence hall	HumanSocial	D	*	217	2,672
Ms	Sampson	HumanSocial	D	*	18	188
YS	Yahoo songs	Rating	B	*	1,625,951	256,804,235
SX	Sexual escorts	Rating	B	*	16,730	50,632

konect.uni-koblenz.de/networks

Polarization

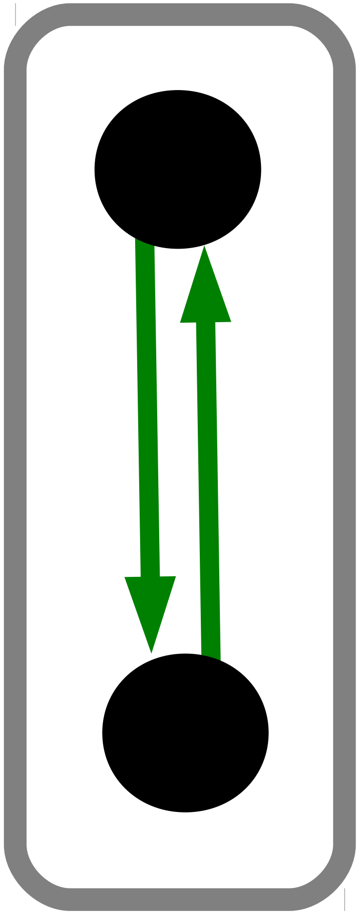
- Slashdot: 23.9% of edges are negative

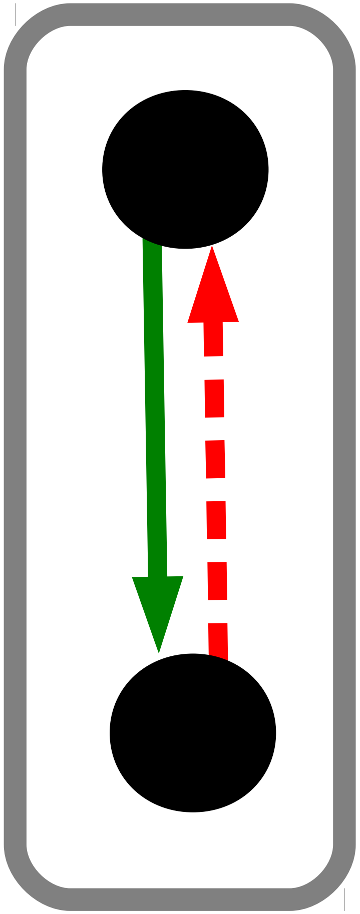


Polarization,
No conflict

Dyadic Conflict (Reciprocity of Valences)

Balance

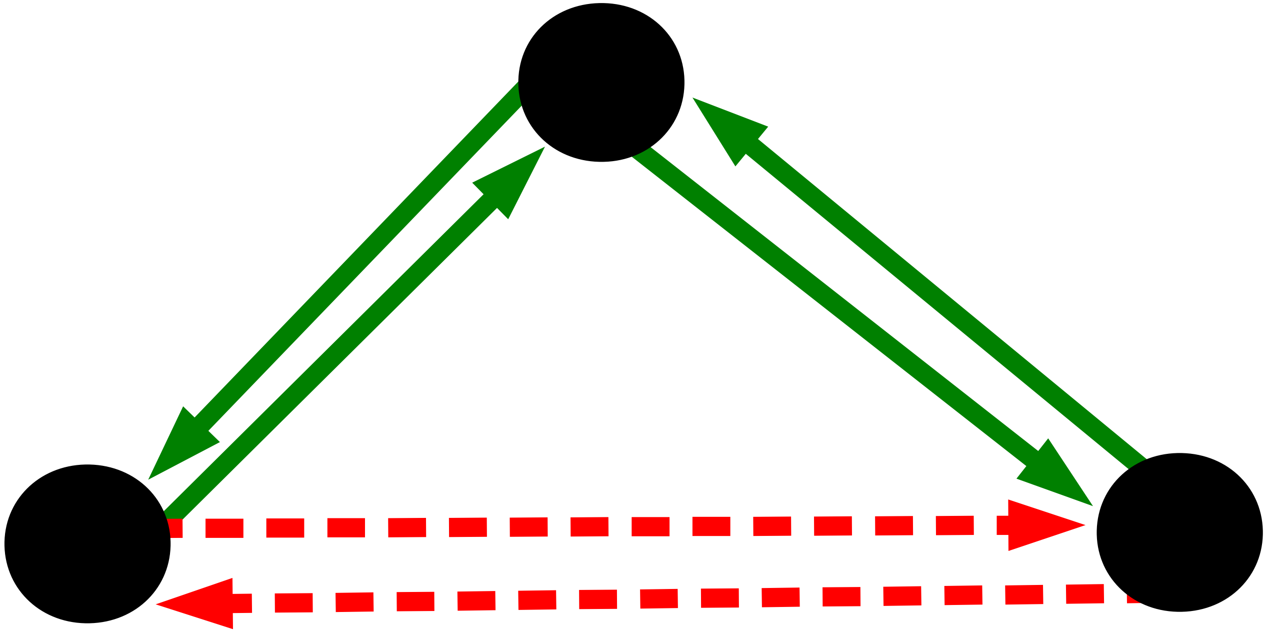




Conflict

Measuring Dyadic Conflict

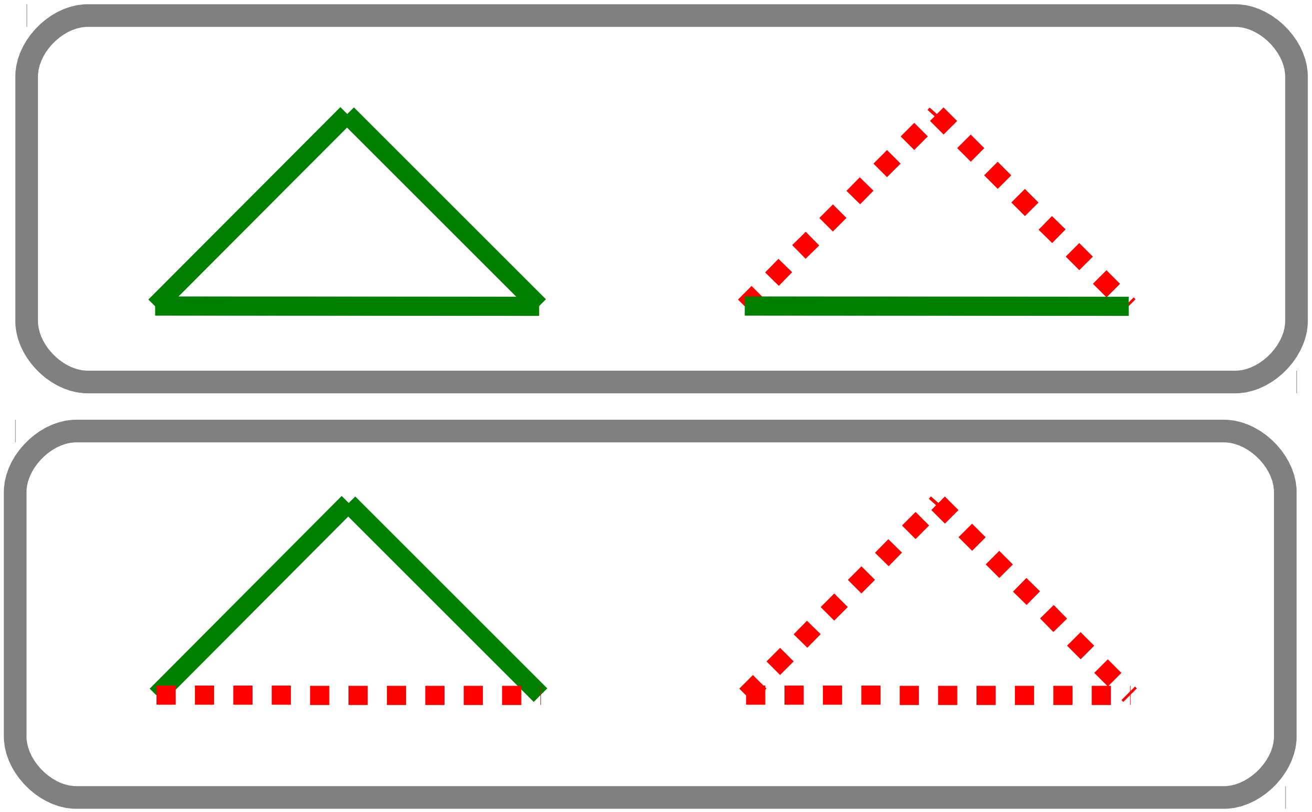
$$C_2 = \frac{\#\text{conflictDyads}}{\#\text{totalDyads}}$$



$C_2 = 0$

Tryadic Conflict (Balance Theory)

Balance



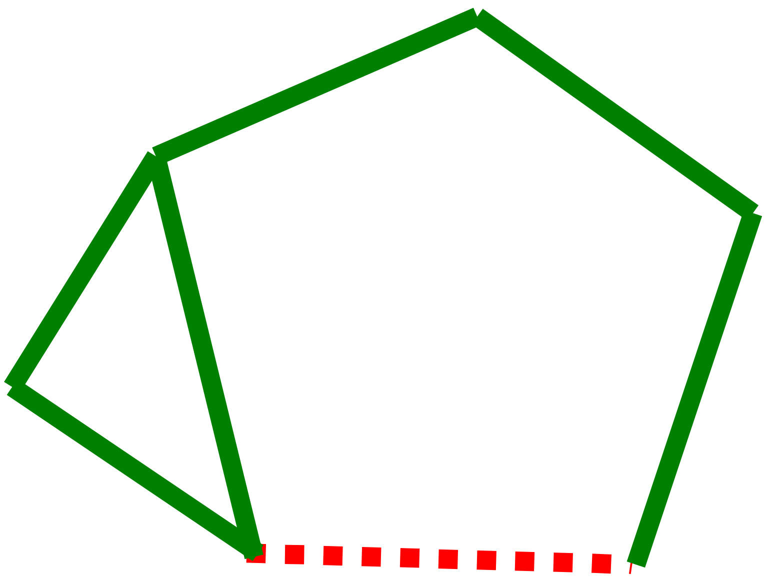
Conflict

(Harary 1953)

Measuring Tryadic Conflict

- Definition:

$$C_3 = \frac{\#negTriangles}{\#totalTriangles}$$

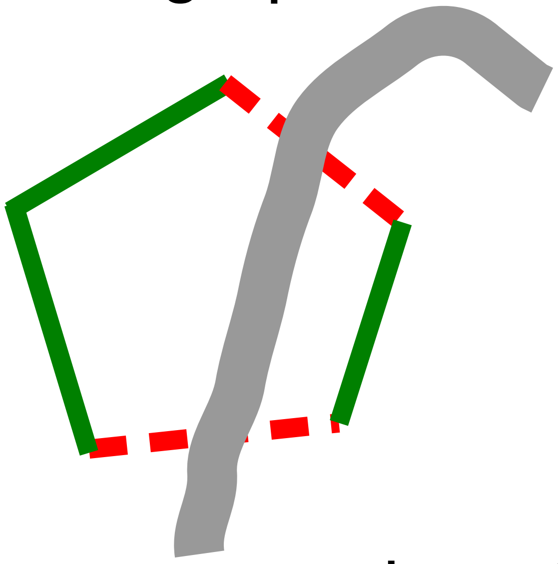


$C_3 = 0$

(cf. Signed relative clustering coefficient, Kunegis et al. 2009)

Balance on Longer Cycles

- Equivalent definitions: a graph is balanced when



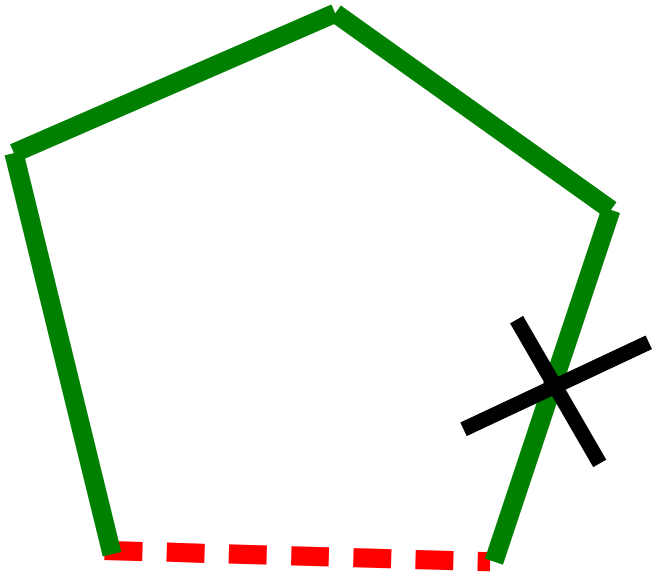
(a) all cycles contain an even number of negative edges

(b) its nodes can be partitioned into two groups such that all positive edges are within each group, and all negative edges connect the two groups

Frustration

- Definition: The minimum number of edges *f* that have to be removed from a signed graph to make the graph balanced.

Example:

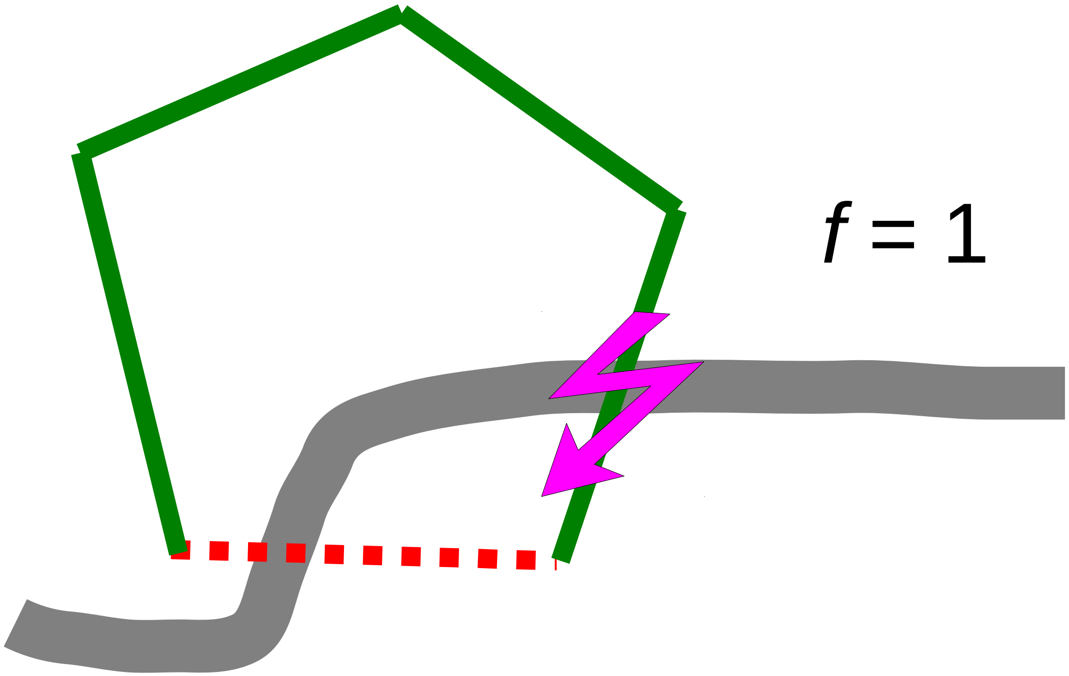


$f = 1$

Frustration (partitioning view)

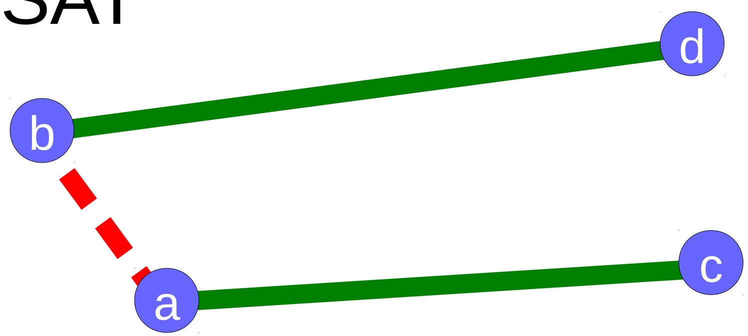
- Definition: minimum number of edges that are *frustrated* (i.e., inconsistent with balance) given any partition of the graph's nodes into two groups.

Example:



Frustration: Computation

- Computation of frustration is equivalent to MAX-2-XORSAT



max (a XOR b) + (¬a XOR c) + (b XOR ¬d) ⋯

- MAX-2-XORSAT is NP complete

- Solution: Relax the problem

see overview in (Facchetti & al. 2011)

Algebraic Formulation

- Let $G = (V, E, \sigma)$ be a signed graph. $\sigma_{uv} = \pm 1$ is the sign of edge (uv).
- Given a partition $V = S \cup T$, let x be the characteristic node-vector:

$$x_u = \begin{cases} +1/2 \text{ when } u \in S \\ -1/2 \text{ when } u \in T \end{cases}$$

- Number of frustrated edges: $\sum_{uv \in E} (x_u - \sigma_{uv} x_v)^2$

Frustration as Minimization

- f is given by the solution to:

$$f^* = \min_x \sum_{uv \in E} (x_u - \sigma_{uv} x_v)^2$$

$$\text{s.t.} \quad \cancel{x \in \{\pm 1/2\}^V}$$

Relaxation

$$\sum_u x_u^2 = |V| / 4$$

$$\Leftrightarrow \|x\| = \sqrt{|V|} / 2$$

Using Matrices

- The quadratic form can be expressed using matrices

$$\sum_{uv \in E} (x_u - \boldsymbol{\sigma}_{\mathbf{uv}}\, x_v)^2 = \frac{1}{2} x^T L\, x$$

where $L \in \mathbb{R}^{V \times V}$ is the matrix given by

$$L_{uv} = -\boldsymbol{\sigma}_{\mathbf{uv}} \text{ when (uv) is an edge}$$

$$L_{uu} = d(u) \text{ is the degree of node u}$$

- $L = D - A$ is the **signed** graph Laplacian

Minimizing Quadratic Forms

$$f^* = \min_x \frac{1}{2} x^T L x$$
$$\text{s.t. } ||x|| = \sqrt{|V|} / 2$$

Rayleigh quotient

$$\frac{8}{|V|} f^* = \min_x \frac{x^T L x}{x^T x}$$

min-max theorem

$$\frac{8}{|V|} f^* = \lambda_{min}[L]$$

$$f^* = \frac{|V|}{8} \lambda_{min}[L]$$

Relative Relaxed Frustration

- Definition: Proportion of edges that have to be removed to make the graph balanced

$$F^* = \frac{f^*}{|E|}$$

$$\boxed{F^* = \frac{|V|}{8\,|E|}\,\lambda_{min}[L]}$$

$$0 \leq F^* \leq \frac{f}{|E|} \leq 1$$

$$\lambda_{min}[L] \leq \frac{8\,|E|}{|V|}$$

Properties of L (Unsigned Graphs)

- L is positive-semidefinite (all $\lambda[L] \geq 0$)
- Multiplicity of $\lambda = 0$ equals number of connected components
- Smallest eigenvalue measures conflict
- Second-smallest eigenvalue measures connectivity (“algebraic connectivity”)

Properties of L (Signed Graphs)

$$L = \sum_{uv \in E} L^{(uv)}$$

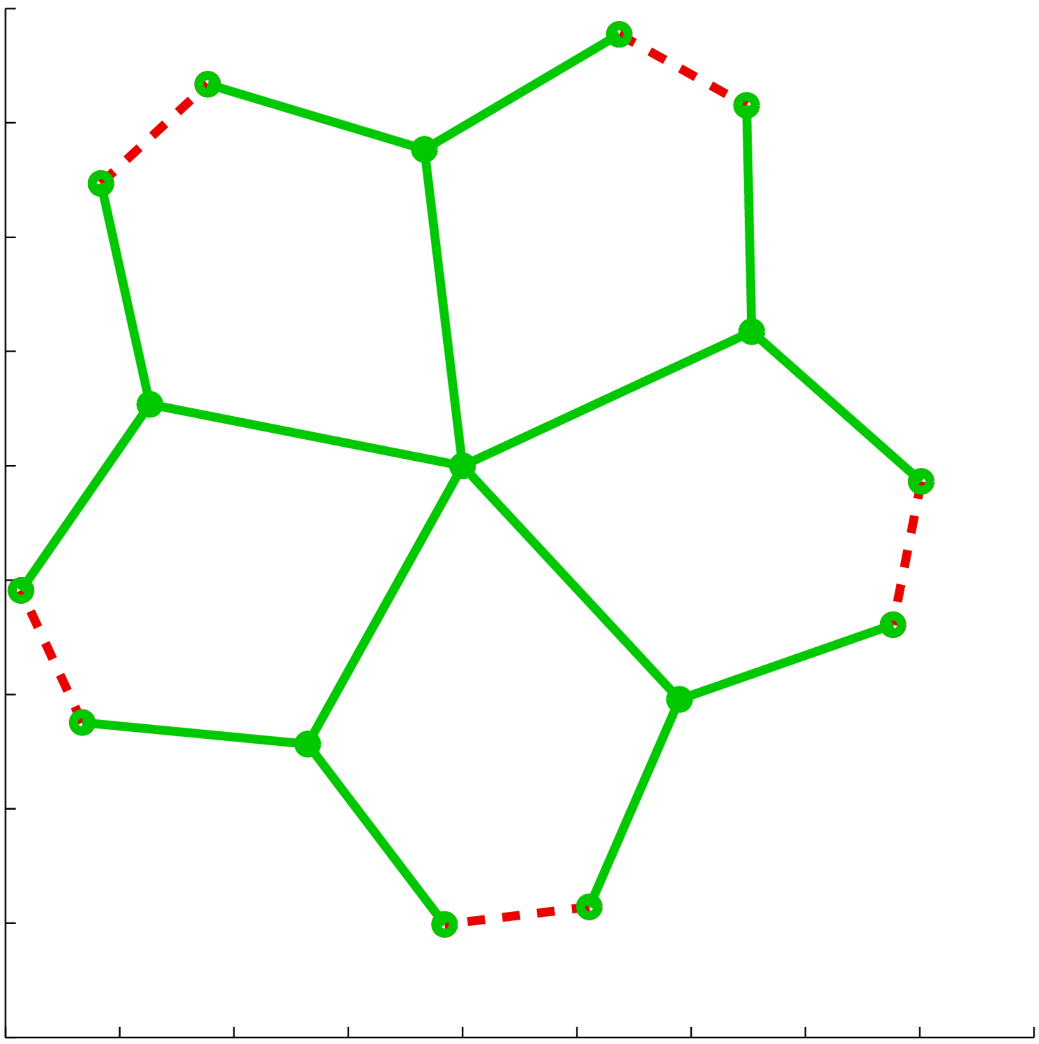
$$L^{(uv)} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{ when } \sigma_{uv} = +1$$

$$L^{(uv)} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ when } \sigma_{uv} = -1$$

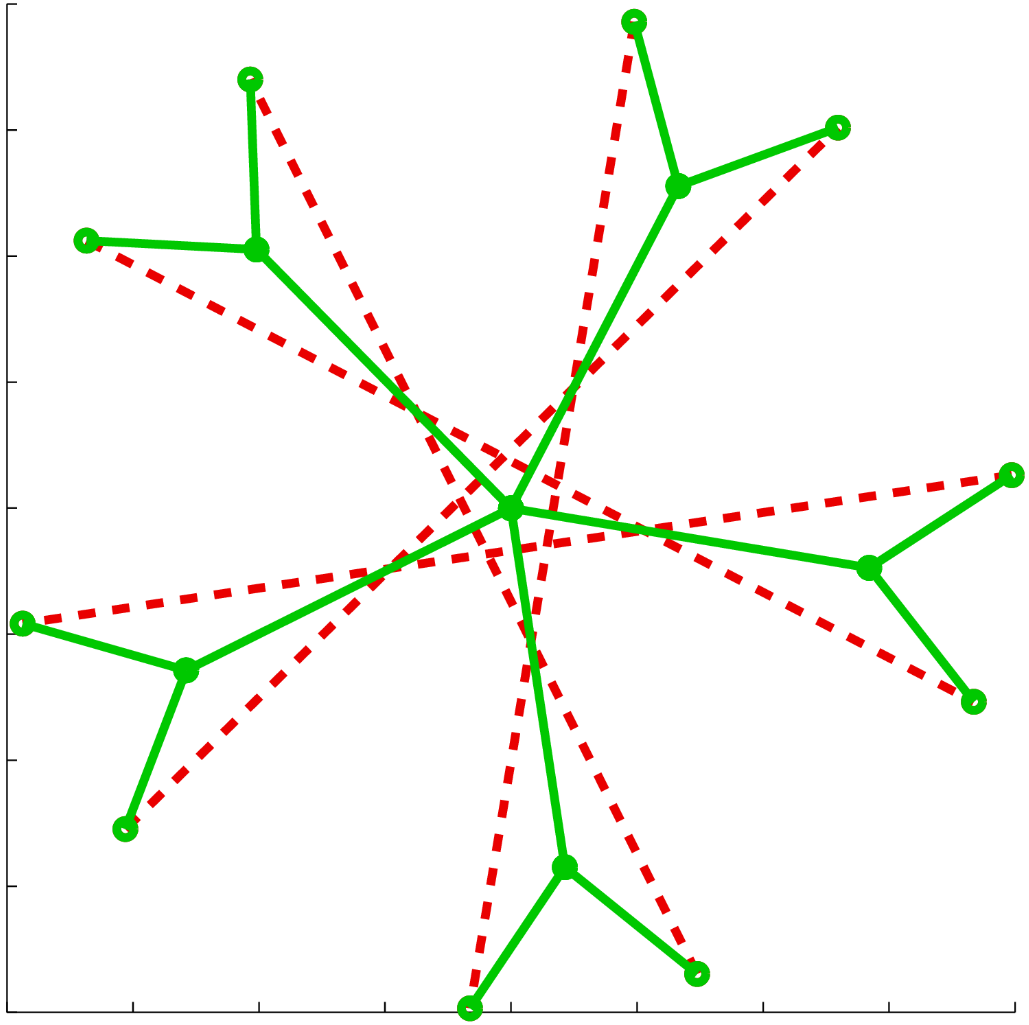
Minimal Eigenvalue of L (Signed Graphs)

- L is positive-semidefinite (all eigenvalues ≥ 0)
- L is positive-definite (all eigenvalues > 0) iff all connected components are unlabanced
  - Proof "$\Rightarrow$": by equivalence by all-positive graph
  - Proof "$\Leftarrow$": by contradiction (eigenvector would be all-zero)

What Unsigned L can Do



What Signed L Can Do



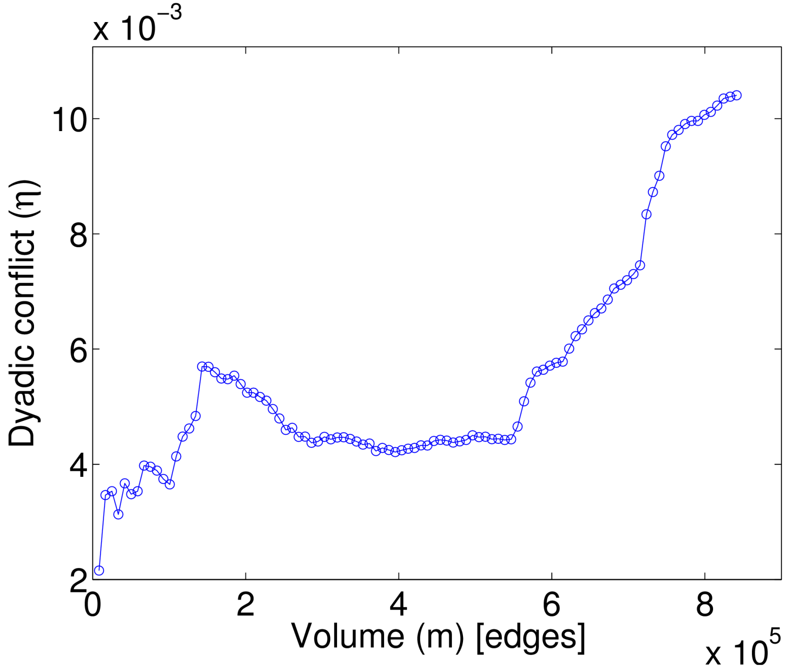
Computing $\lambda_{min}[L]$

- Sparse LU decomposition + inverse power iteration:

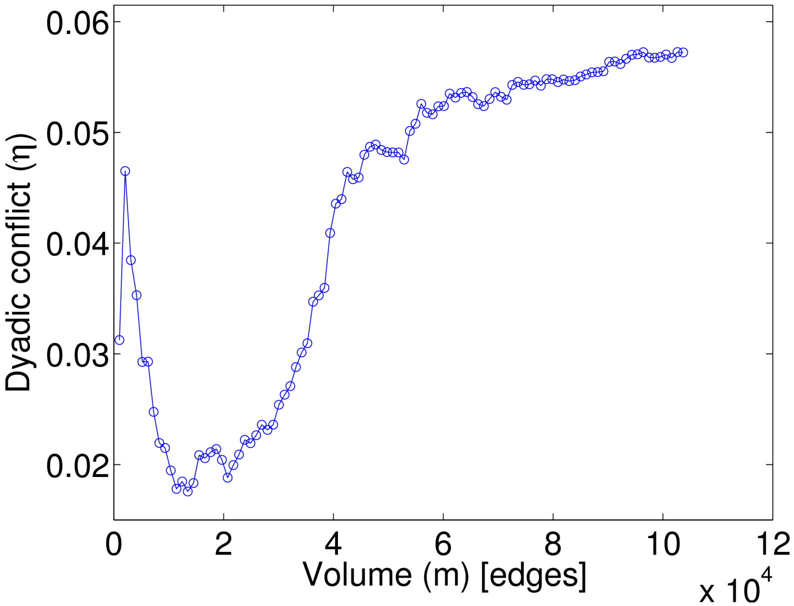
  $O(|V|^2)$ memory, but then very fast

```
% Matlab pseudocode
[ X Y ] = sparse_lu(L);
[ U D ] = eigs(@(x)(Y \ X \ x), k, 'sm');
```

Temporal Analysis of C_2

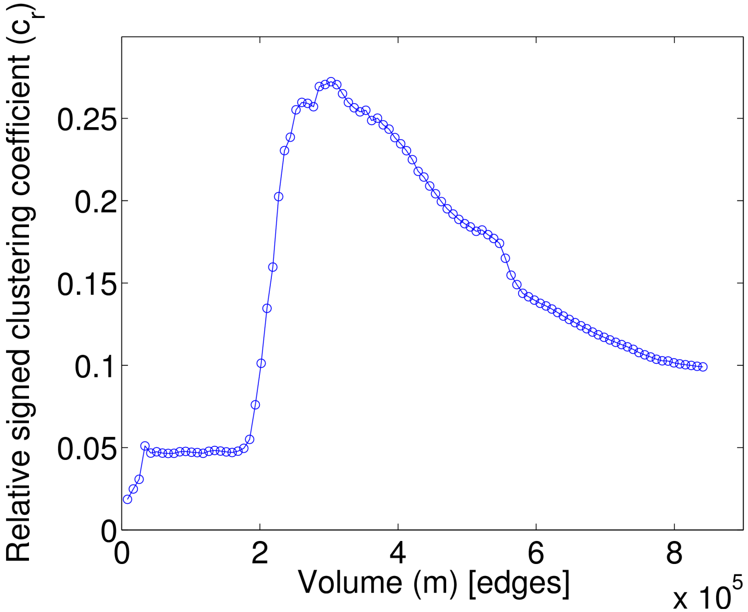


Epinions ratings

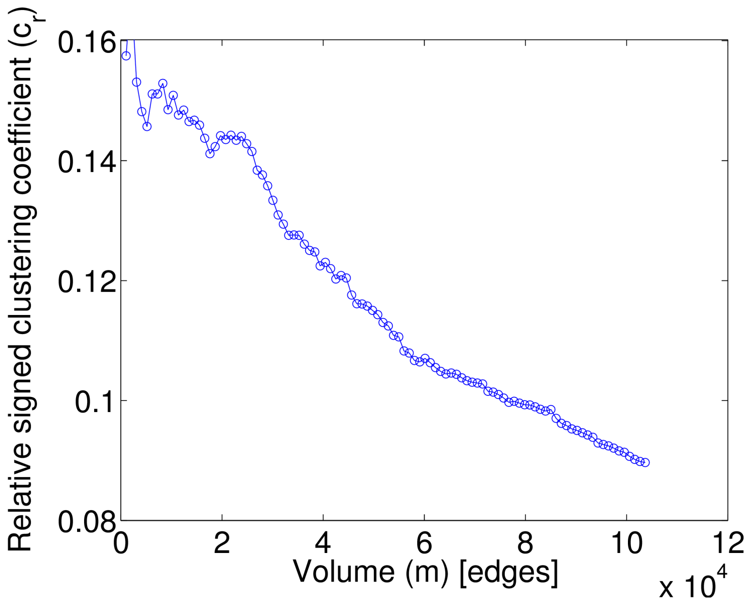


Wikipedia elections

Temporal Analysis of C₃

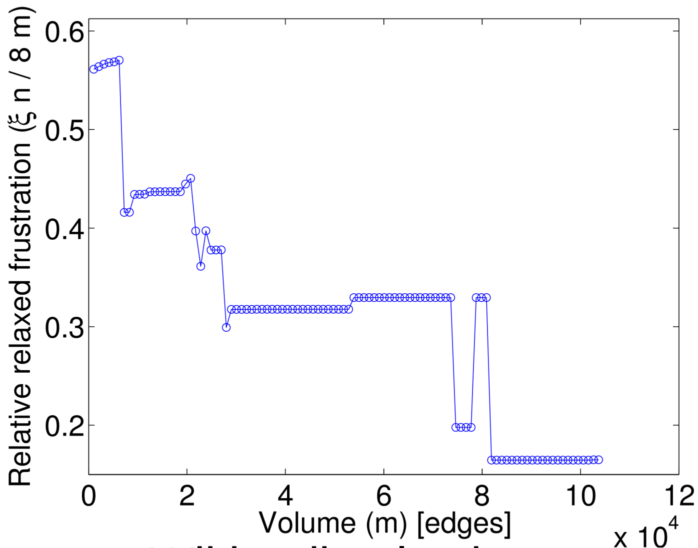


Epinions ratings

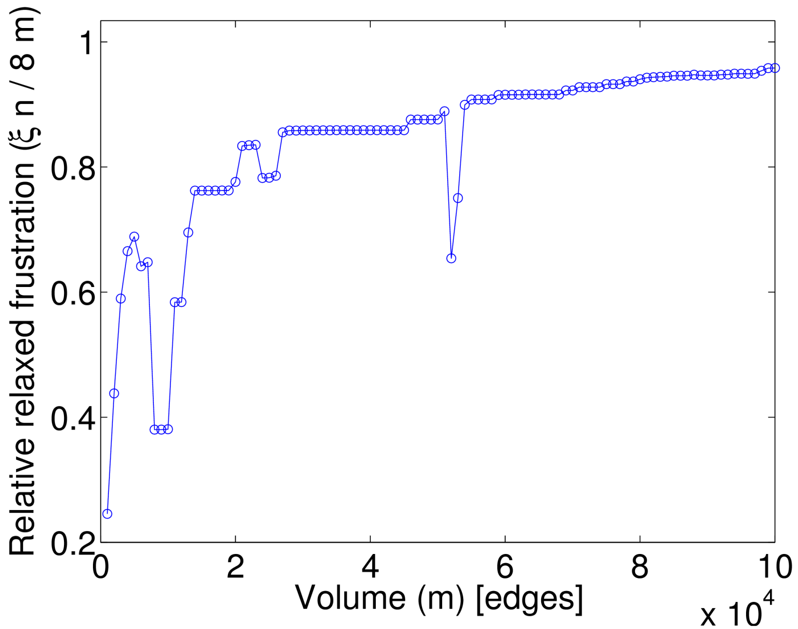


Wikipedia elections

F* over Time

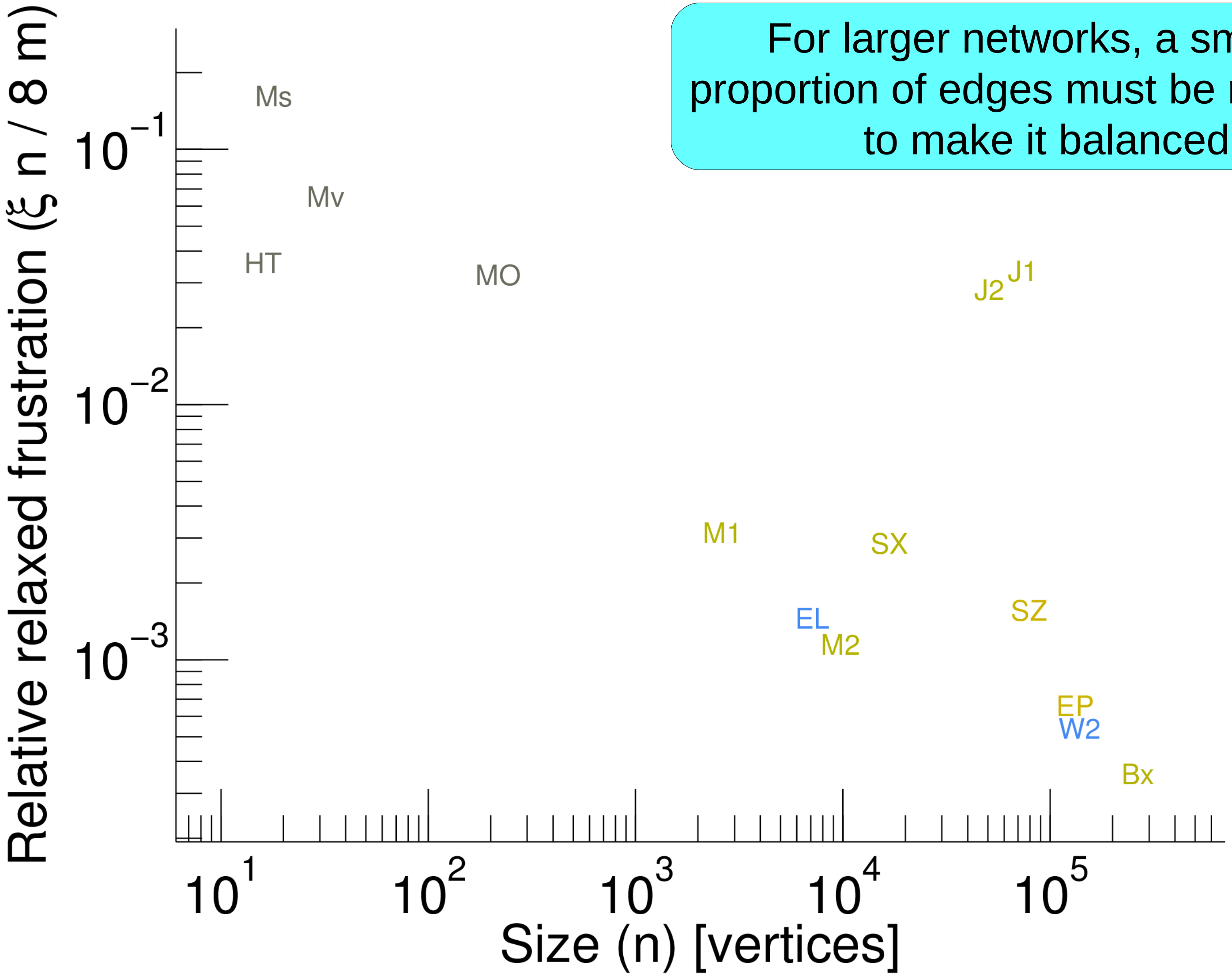


Wikipedia elections



MovieLens 100k

Cross-Dataset Comparison of F*



For larger networks, a smaller proportion of edges must be removed to make it balanced

Merci

konect.uni-koblenz.de

Contact: Jérôme Kunegis

Université Koblenz-Landau

<kunegis@uni-koblenz.de> @kunegis

We want more datasets with negative edges, and timestamps.

- In particular: with changing and/or dissapearing edges