



# Algebraic Graph-theoretic Measures of Conflict

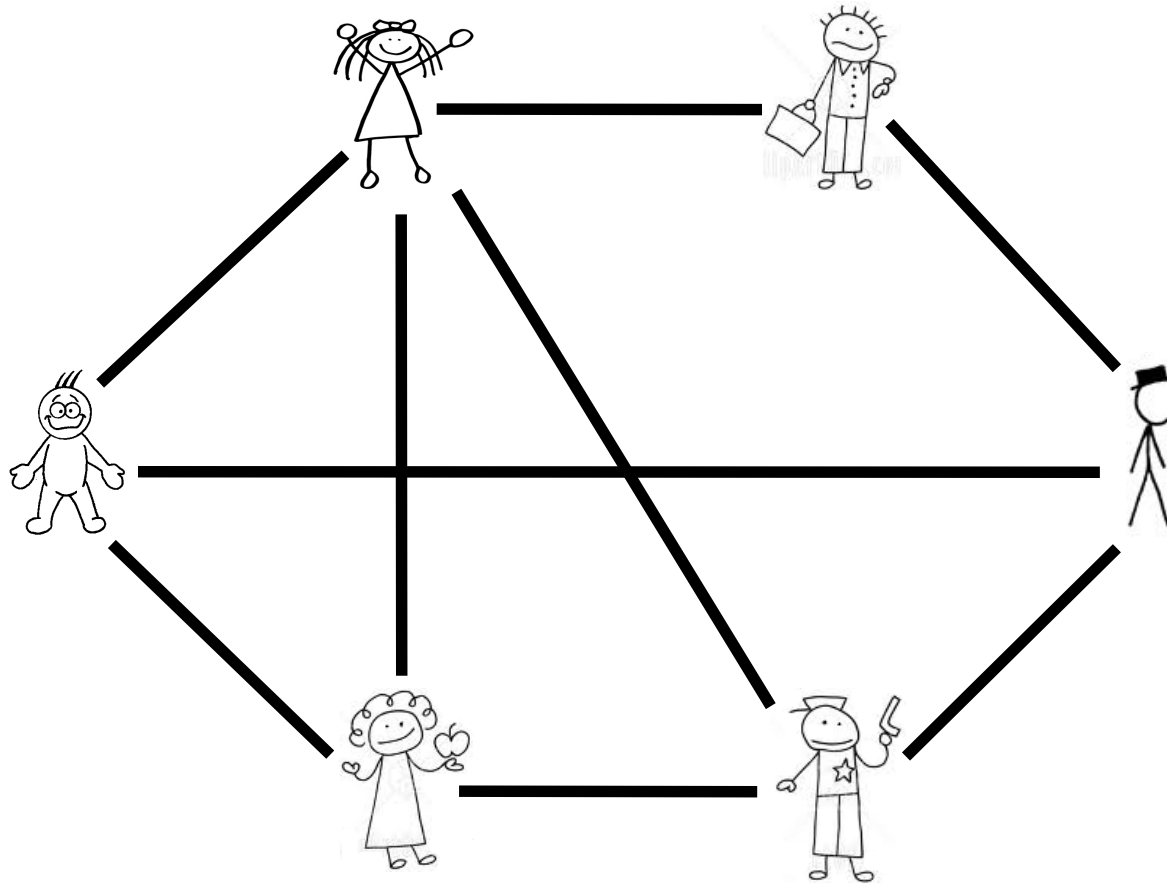
Jérôme Kunegis

Based on work performed in collaboration with Christian Bauckhage, Andreas Lommatzsch, Stephan Spiegel, Jürgen Lerner, Fariba Karimi and Christoph Carl Kling

Journée Graphes et Systèmes Sociaux (JGSS), March 18, 2016, Avignon



# Social Network



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### Hardware: Sony Recalls 73,000 Vaio Laptops Due To Burn Worry

Posted by [CmdrTaco](#) on Thursday September 04, @11:43AM  
from the moretofear dept.

alphadogg writes

"Sony is recalling 73,000 Vaio TZ laptops because of a possible manufacturing defect that may cause them to overheat, the U.S. Consumer Product Safety Commission said Thursday. The recall relates to a problem with wiring near the computer's hinge, which could short-circuit and overheat in c  
circumstances, [perhaps burning the user](#). One person has suffered a minor burn as a result of the late defect, and Sony has received 15 other reports of overheating computers, according to the Commissi

► hardware, sony (tagging beta)

[Read More...](#) | [hardware.slashdot.org](#)

SOI

32 com

### News: Insects May Have Had a Hand In Dinosaur Extinction

Posted by [CmdrTaco](#) on Thursday September 04, @11:00AM  
from the sodid-mycoking dept.

eldavojohn writes

"Everyone's got their favorite theories of Dinosaur extinction but new speculation is rampant in [a book](#) cause to believe it may have been [disease carrying insects](#). Due to the length of their slow and eventua  
(the "K-T Boundary"), it is argued that this would more likely be attributed to the spread of disease and  
parasitic insects like ticks or biting flies. Are our immune systems the only reason any animals survived

► science, earth (tagging beta)

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### Chrome iPhone (Score:5, Funny)

by [oldhack \(1037484\)](#) on Wednesday September 03, @09:55PM (#248

Stick Chrome with iPhone and you can run them stories to fill up a

Reply to This

### Re:Chrome iPhone (Score:5, Funny)

by [commodoresloat \(172735\)](#) \* on Wednesday September 03, @10:0

should be easy for google to do coz all they have to do to get th

Reply to This

Parent

### Re:Firefox Damage Control Is More Than Enough (Score:5, Funny)

by Anonymous Coward on Wednesday September 03, @10:49

Forget the iPhone. The **AM**ount of **dA**mage **conT**ROL a  
the Net.

There. You said it all there.

Reply to This

Parent

### Re:Firefox Damage Control Is More Than Enough (Score:5, Funny)

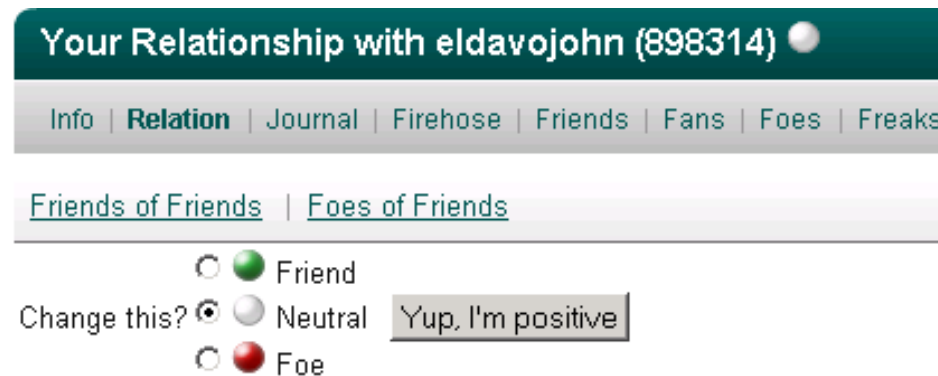
by [mweather \(1089505\)](#) on Wednesday September 03, @10:49

Konqueror has a windows port, too.

# The Slashdot Zoo

Slashdot Zoo: Tag users as *friends* and *foes*

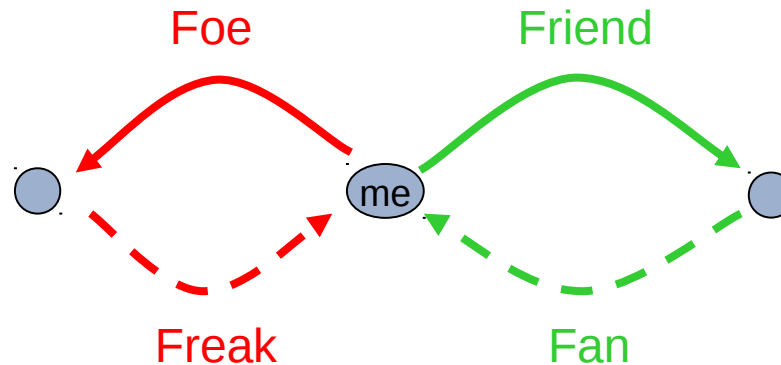
Graph has *two* types of edges: friendship and enmity



The screenshot shows a web interface for managing a relationship with a user named 'eldavojohn (898314)'. At the top is a dark green header with the title 'Your Relationship with eldavojohn (898314)' and a small grey circle icon. Below this is a light grey navigation bar with links: 'Info', 'Relation' (highlighted in dark green), 'Journal', 'Firehose', 'Friends', 'Fans', 'Foes', and 'Freaks'. Underneath is another light grey bar with links: 'Friends of Friends' and 'Foes of Friends'. The main content area shows three radio button options for the relationship: 'Friend' (with a green circle), 'Neutral' (with a grey circle), and 'Foe' (with a red circle). The 'Friend' option is selected. To the left of these options is the text 'Change this?'. To the right is a text input field containing the text 'Yup, I'm positive'.

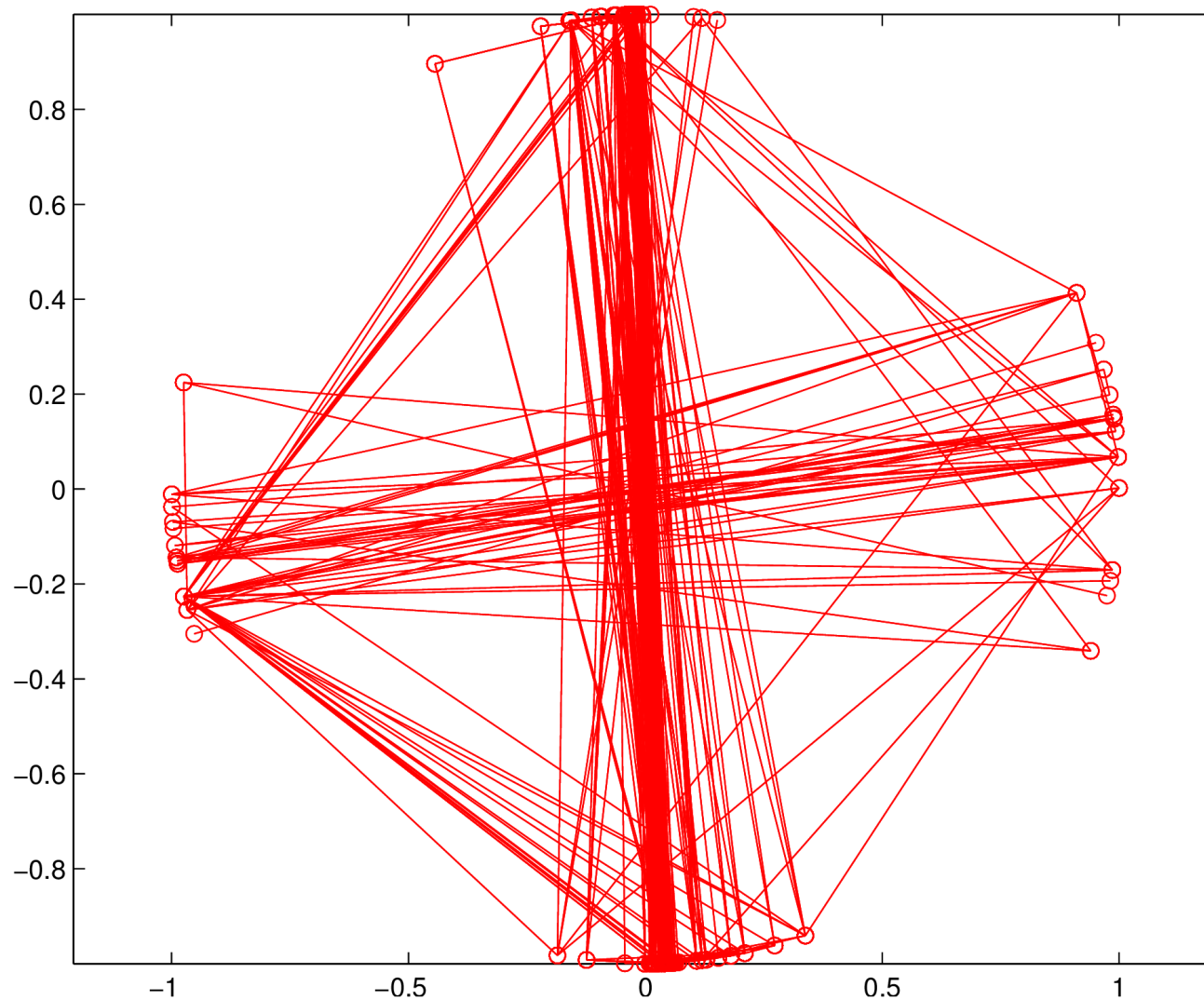
# Signed Directed Social Network

You are the fan of your friends and the freak of your foes.



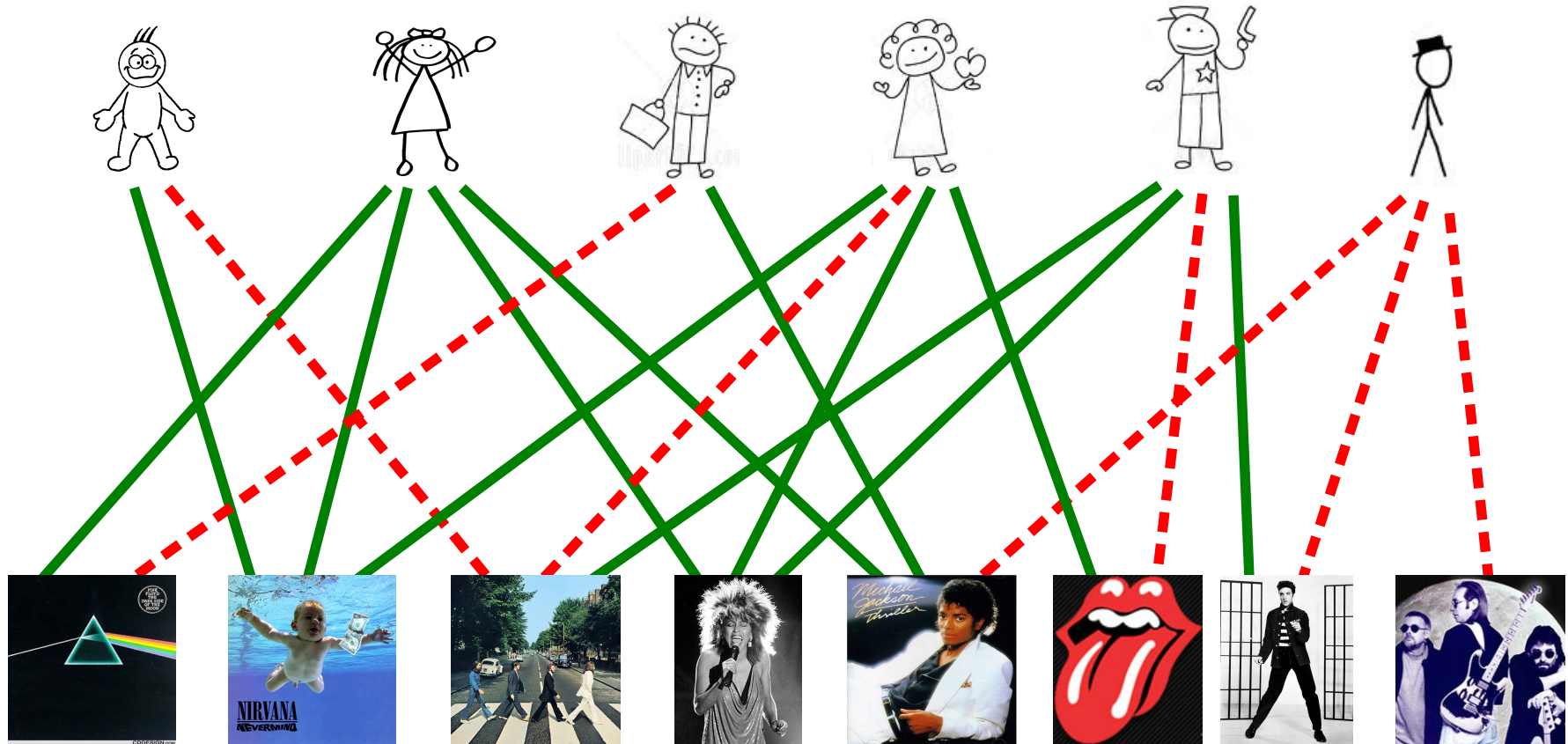
The resulting graph is sparse, square, asymmetric and has signed edge weights.

# Wikipedia Edit Wars





# Signed Bipartite Networks



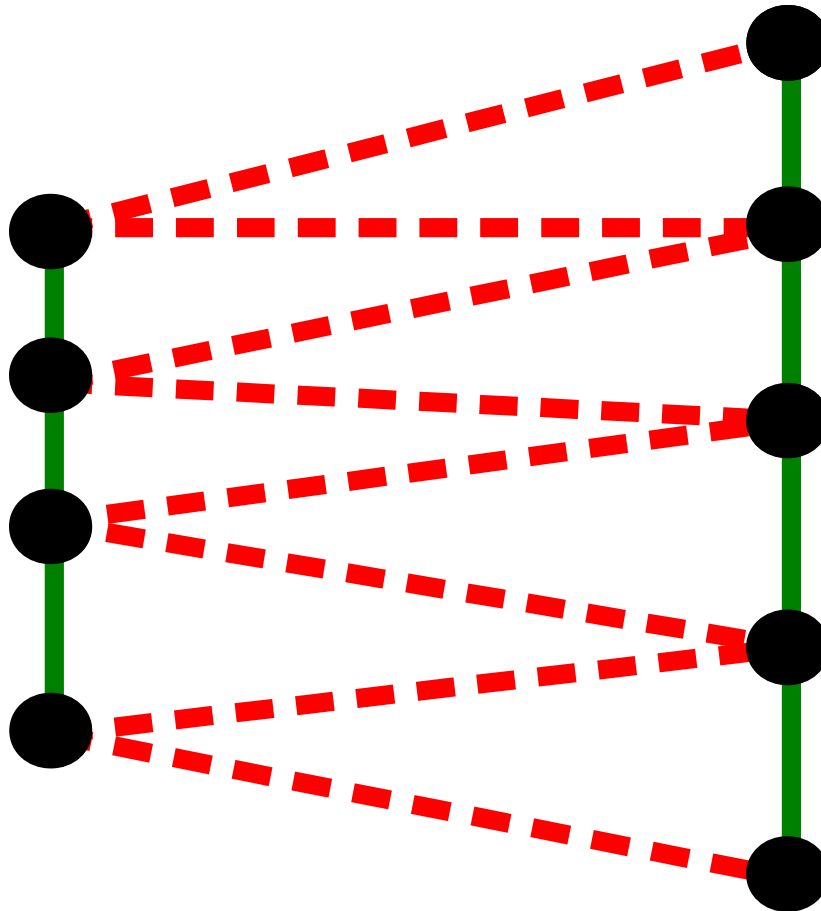
# Other Signed Networks

<b>Mv</b>	Dutch college	HumanSocial	D $\pm$		32	3,062		
<b>EP</b>	Epinions trust	Social	D $\pm$		131,828	841,372		
<b>HT</b>	Highland tribes	HumanSocial	U $\pm$		16	58		
<b>SZ</b>	Slashdot Zoo	Social	D $\pm$		79,120	515,397		
<b>CO</b>	Wikipedia conflict	OnlineContact	U $\pm$		118,100	2,917,785		
<b>EL</b>	Wikipedia elections	OnlineContact	D $\pm$		7,118	103,675		
<b>W2</b>	WikiSigned	OnlineContact	D $\pm$		138,592	740,397		
<b>AR</b>	Amazon ratings	Rating	B $\star$		3,376,972	5,838,041		
<b>Bx</b>	BookCrossing (ratings)	Rating	B $\star$		263,757	433,652		
<b>ER</b>	Epinions product ratings	Rating	B $\star$		876,252	13,668,320		
<b>Fr</b>	FilmTipSet	Rating	B $\star$		144,671	19,554,219		
<b>J1</b>	Jester 100	Rating	B $\star$		73,521	4,136,360		
<b>J2</b>	Jester 150	Rating	B $\star$		50,832	1,728,847		
<b>LI</b>	Libimseti.cz	Social	D $\star$		220,970	17,359,346		
<b>M1</b>	MovieLens 100k	Rating	B $\star$		2,625	100,000		
<b>M3</b>	MovieLens 10M	Rating	B $\star$		80,555	10,000,054		
<b>M2</b>	MovieLens 1M	Rating	B $\star$		9,746	1,000,209		
<b>NX</b>	Netflix	Rating	B $\star$		497,959	100,480,507		
<b>MO</b>	Residence hall	HumanSocial	D $\star$		217	2,672		
<b>Ms</b>	Sampson	HumanSocial	D $\star$		18	188		
<b>YS</b>	Yahoo songs	Rating	B $\star$		1,625,951	256,804,235		
<b>SX</b>	Sexual escorts	Rating	B $\star$		16,730	50,632		



# Polarization

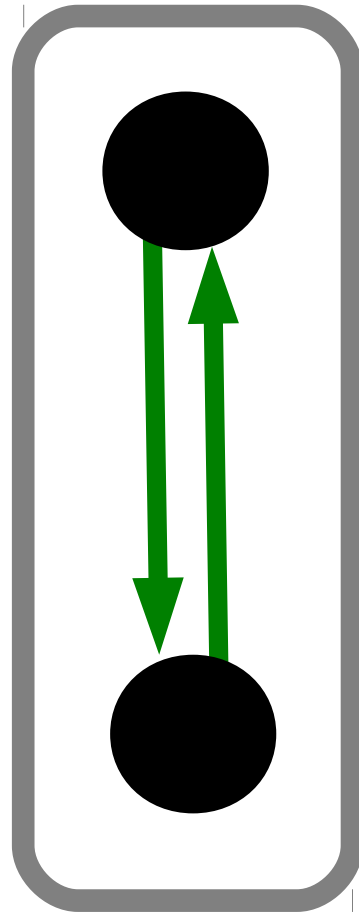
- Slashdot: 23.9% of edges are negative



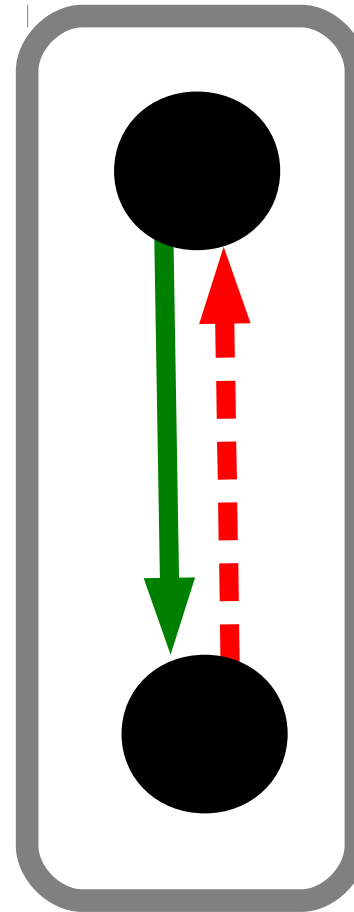
Polarization,  
No conflict

# Dyadic Conflict (Reciprocity of Valences)

Balance

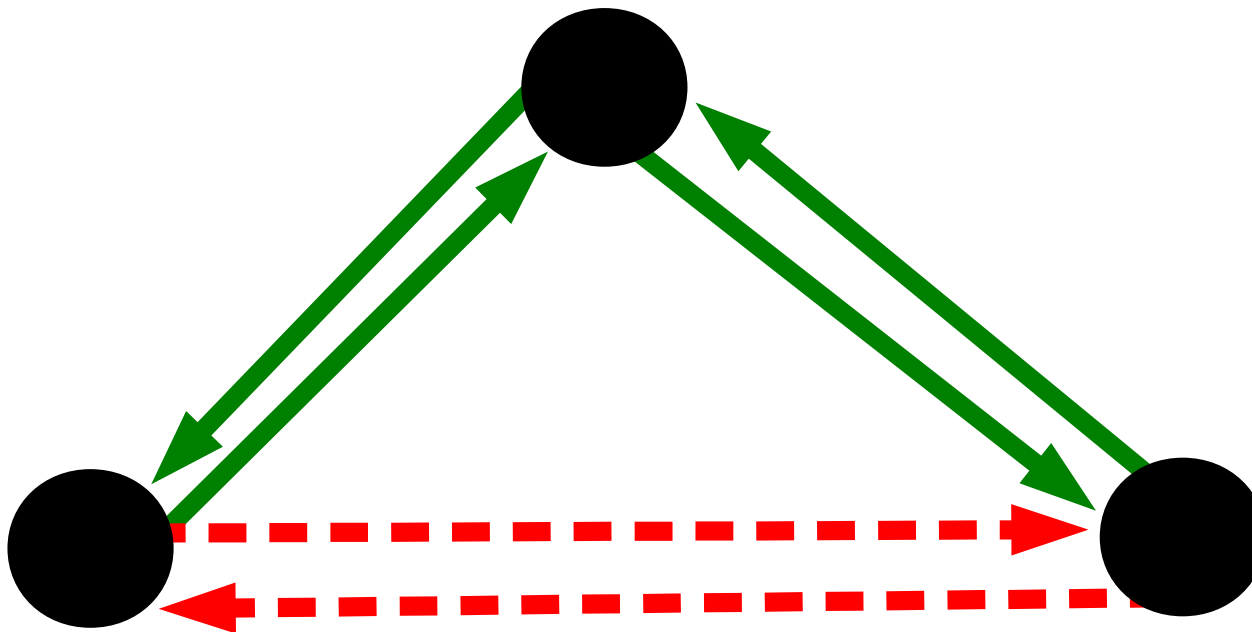


Conflict



# Measuring Dyadic Conflict

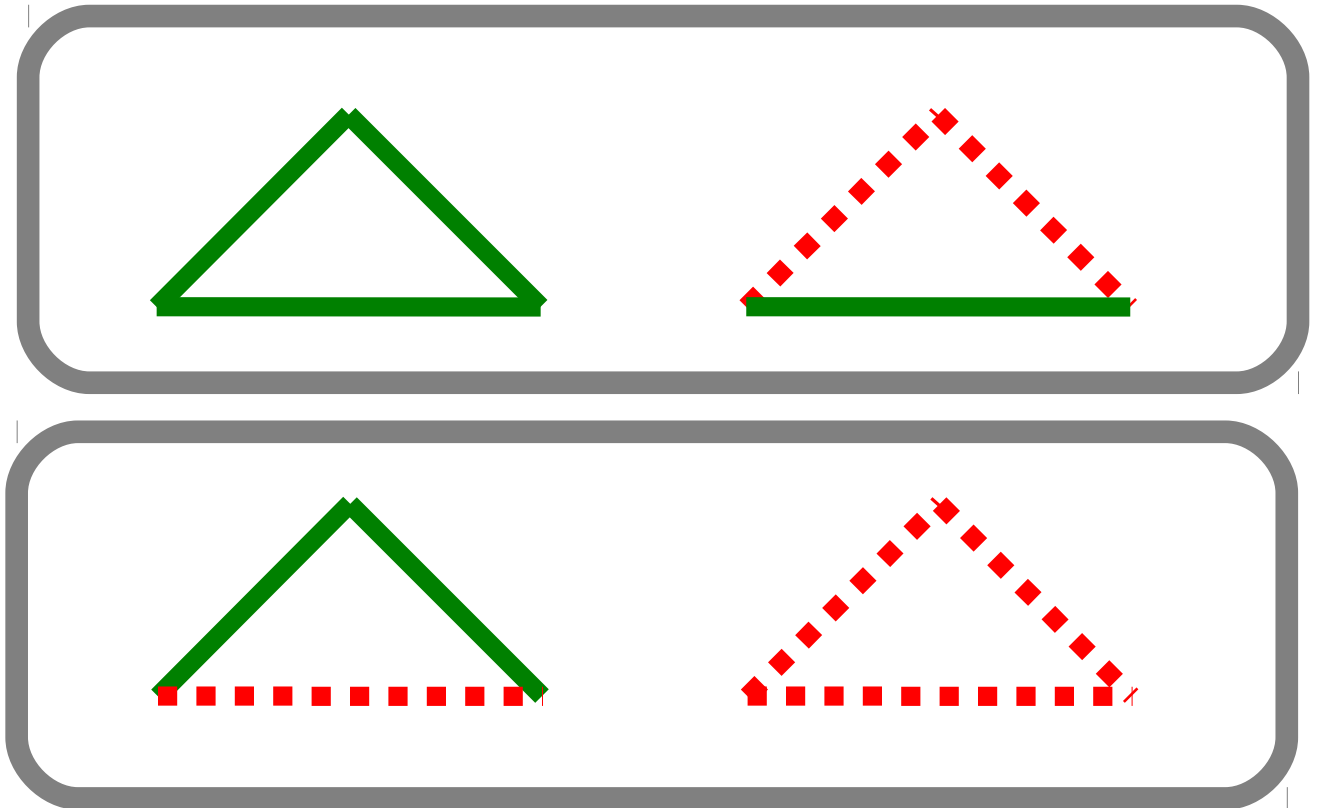
$$C_2 = \frac{\text{\#conflictDyads}}{\text{\#totalDyads}}$$



$$C_2 = 0$$

# Tryadic Conflict (Balance Theory)

Balance



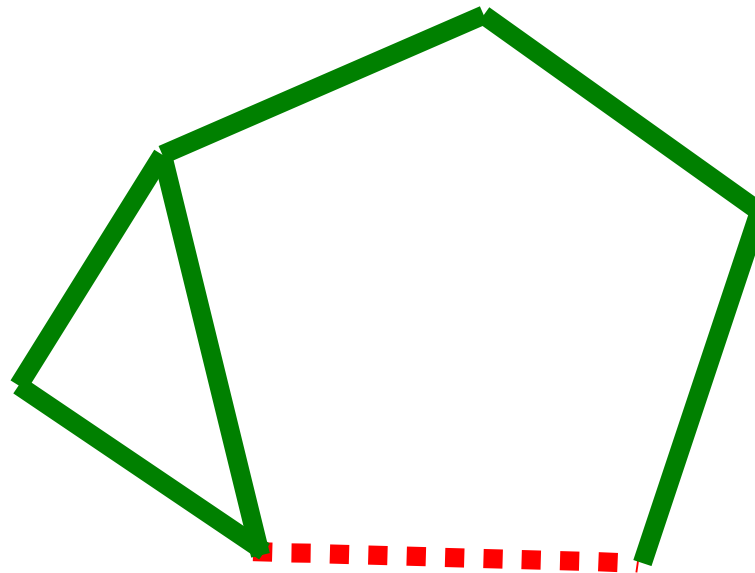
Conflict

(Harary 1953)

# Measuring Tryadic Conflict

- Definition:

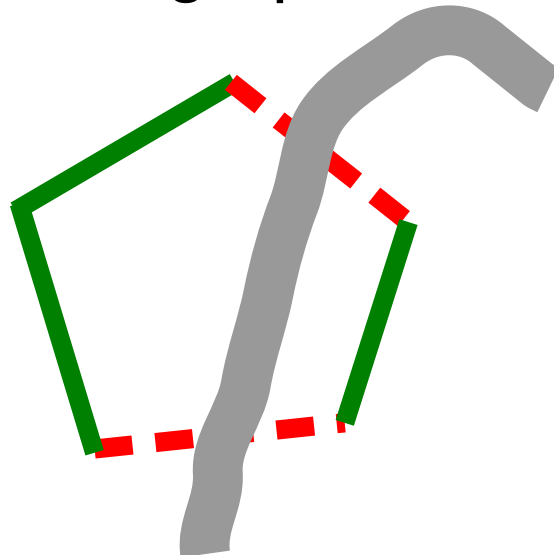
$$C_3 = \frac{\text{\#negTriangles}}{\text{\#totalTriangles}}$$



(cf. Signed relative clustering coefficient, Kunegis et al. 2009)

# Balance on Longer Cycles

- Equivalent definitions: a graph is balanced when



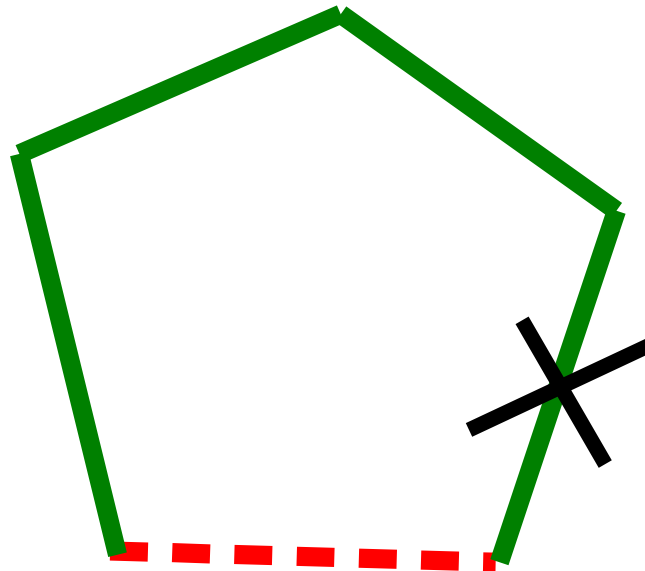
- (a) all cycles contain an even number of negative edges
- (b) its nodes can be partitioned into two groups such that all positive edges are within each group, and all negative edges connect the two groups



# Frustration

- Definition: The minimum number of edges  $f$  that have to be removed from a signed graph to make the graph balanced.

Example:

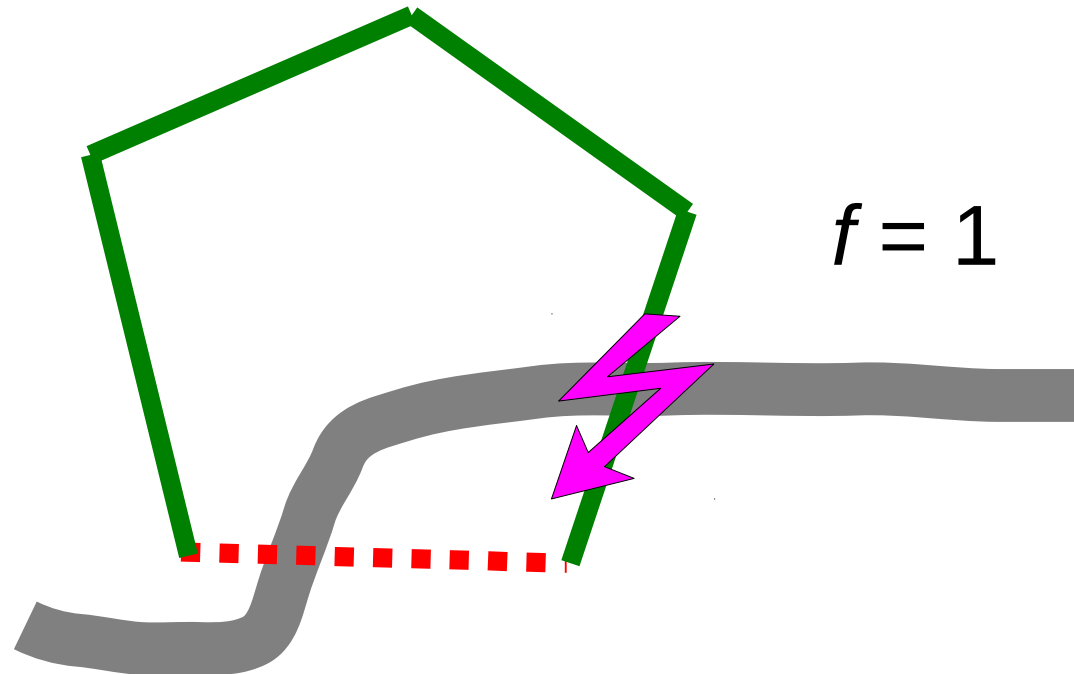


$$f = 1$$

# Frustration (partitioning view)

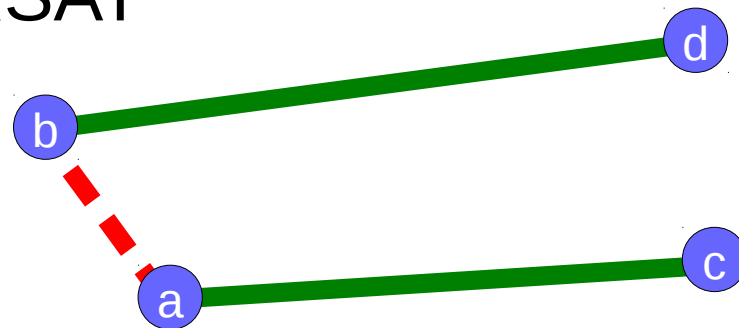
- Definition: minimum number of edges that are *frustrated* (i.e., inconsistent with balance) given any partition of the graph's nodes into two groups.

Example:



# Frustration: Computation

- Computation of frustration is equivalent to MAX-2-XORSAT



$$\max \underbrace{(a \text{ XOR } b)}_{\text{dashed red}} + \underbrace{(\neg a \text{ XOR } c)}_{\text{solid green}} + \underbrace{(b \text{ XOR } \neg d)}_{\text{solid green}} \dots$$

- MAX-2-XORSAT is NP complete
- Solution: Relax the problem

see overview in (Facchetti & al. 2011)

# Algebraic Formulation

- Let  $G = (V, E, \sigma)$  be a signed graph.  $\sigma_{uv} = \pm 1$  is the sign of edge  $(uv)$ .
- Given a partition  $V = S \cup T$ , let  $x$  be the characteristic node-vector:

$$x_u = \begin{cases} +1/2 & \text{when } u \in S \\ -1/2 & \text{when } u \in T \end{cases}$$

- Number of frustrated edges:  $\sum_{uv \in E} (x_u - \sigma_{uv} x_v)^2$

# Frustration as Minimization

- $f$  is given by the solution to:

$$f^* = \min_x \sum_{uv \in E} (x_u - \sigma_{uv} x_v)^2$$

$$\text{s.t. } \cancel{x \in \{\pm 1/2\}^V}$$

$$\sum_u x_u^2 = |V| / 4$$

$$\Leftrightarrow \|x\| = \sqrt{|V|} / 2$$



Relaxation

# Using Matrices

- The quadratic form can be expressed using matrices

$$\sum_{uv \in E} (x_u - \sigma_{uv} x_v)^2 = \frac{1}{2} x^T L x$$

where  $L \in \mathbb{R}^{V \times V}$  is the matrix given by

$L_{uv} = -\sigma_{uv}$  when  $(uv)$  is an edge

$L_{uu} = d(u)$  is the degree of node  $u$

- $L = D - A$  is the **signed** graph Laplacian



# Minimizing Quadratic Forms

$$f^* = \min_x \frac{1}{2} x^T L x$$

s.t.  $\|x\| = \sqrt{|V|} / 2$

$$\frac{8}{|V|} f^* = \min_x \frac{x^T L x}{x^T x}$$

Rayleigh  
quotient

$$\frac{8}{|V|} f^* = \lambda_{\min}[L]$$

min-max  
theorem

$$f^* = \frac{|V|}{8} \lambda_{\min}[L]$$

# Relative Relaxed Frustration

- Definition: Proportion of edges that have to be removed to make the graph balanced

$$F^* = \frac{f^*}{|E|}$$

$$F^* = \frac{|V|}{8 |E|} \lambda_{\min}[L]$$

$$0 \leq F^* \leq \frac{f}{|E|} \leq 1$$

$$\lambda_{\min}[L] \leq \frac{8 |E|}{|V|}$$

# Properties of $L$ (Unsigned Graphs)

- $L$  is positive-semidefinite (all  $\lambda[L] \geq 0$ )
- Multiplicity of  $\lambda = 0$  equals number of connected components
- Smallest eigenvalue measures conflict
- Second-smallest eigenvalue measures connectivity (“algebraic connectivity”)

# Properties of L (Signed Graphs)

$$L = \sum_{uv \in E} L^{(uv)}$$

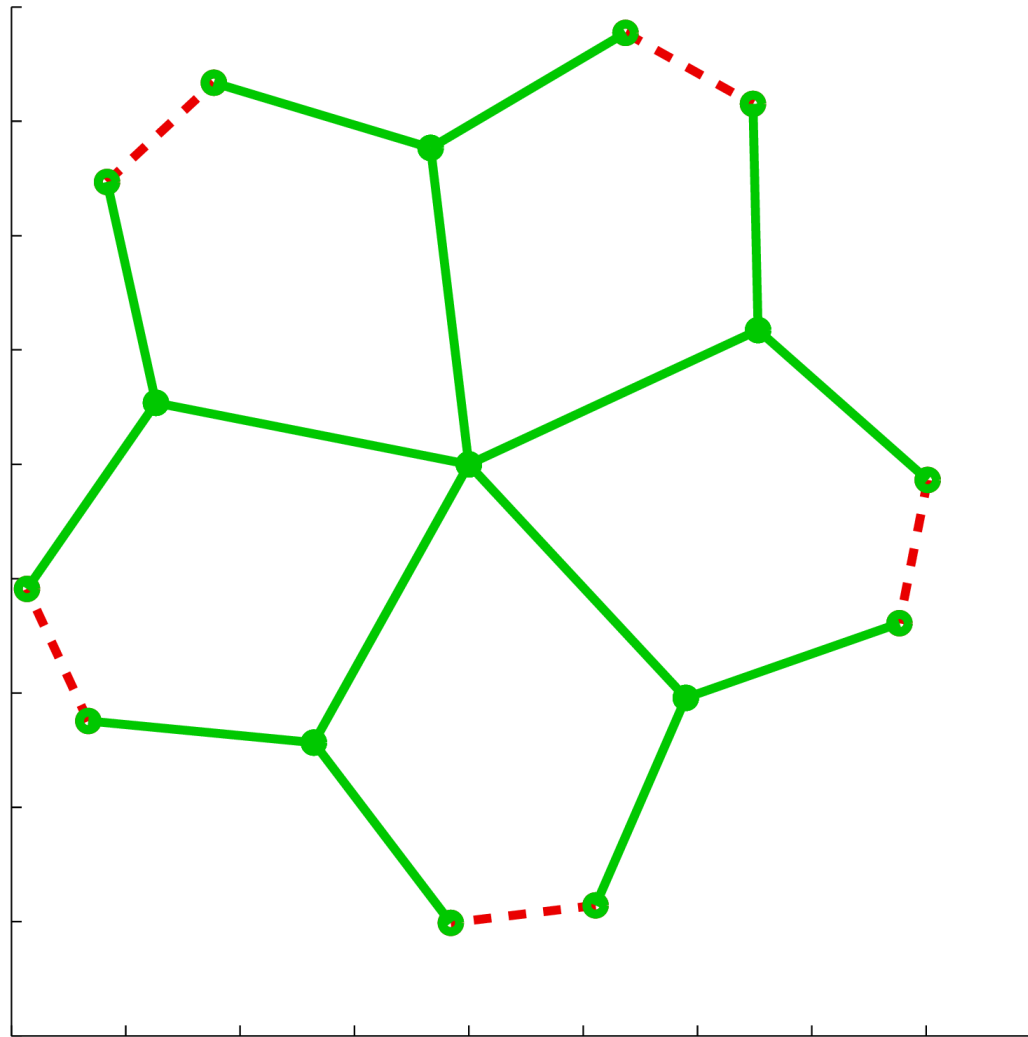
$$L^{(uv)} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{ when } \sigma_{uv} = +1$$

$$L^{(uv)} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ when } \sigma_{uv} = -1$$

# Minimal Eigenvalue of $L$ (Signed Graphs)

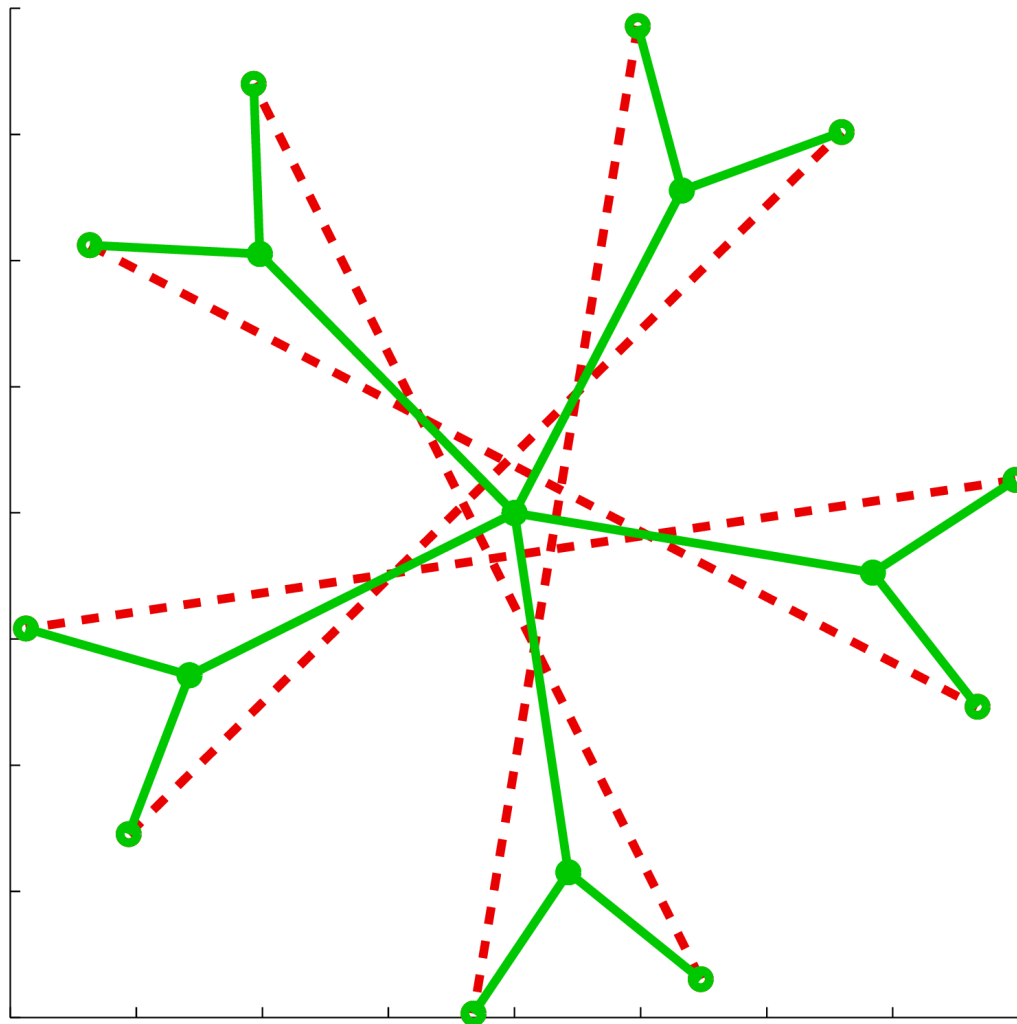
- $L$  is positive-semidefinite (all eigenvalues  $\geq 0$ )
- $L$  is positive-definite (all eigenvalues  $> 0$ ) iff all connected components are unlabanced
  - Proof “ $\Rightarrow$ ”: by equivalence by all-positive graph
  - Proof “ $\Leftarrow$ ”: by contradiction (eigenvector would be all-zero)

# What Unsigned L can Do





# What Signed L Can Do



# Computing $\lambda_{\min}[L]$

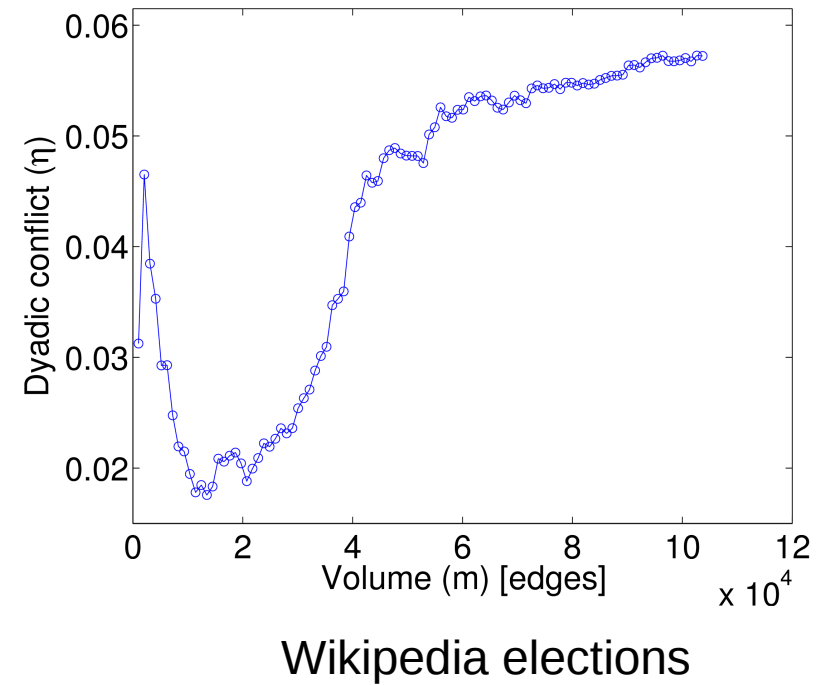
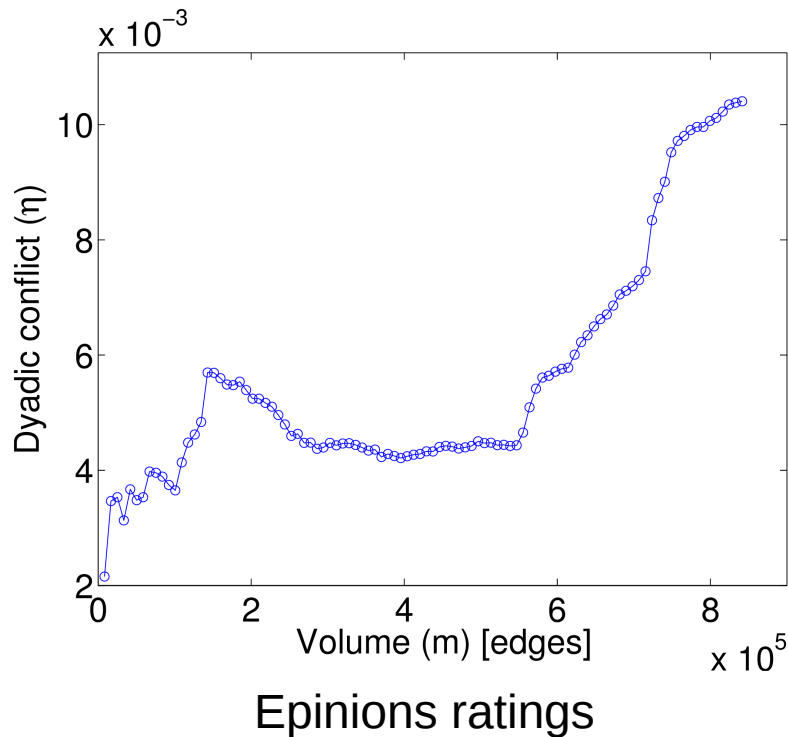
- Sparse LU decomposition + inverse power iteration:  
 $O(|V|^2)$  memory, but then very fast

% Matlab pseudocode

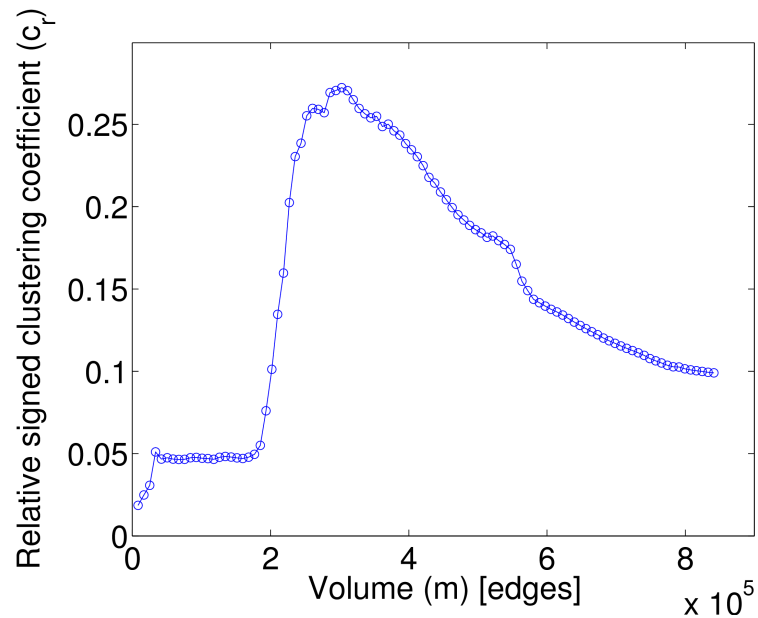
```
[ X Y ] = sparse_lu(L);
```

```
[ U D ] = eigs(@(x)(Y \ X \ x), k, 'sm');
```

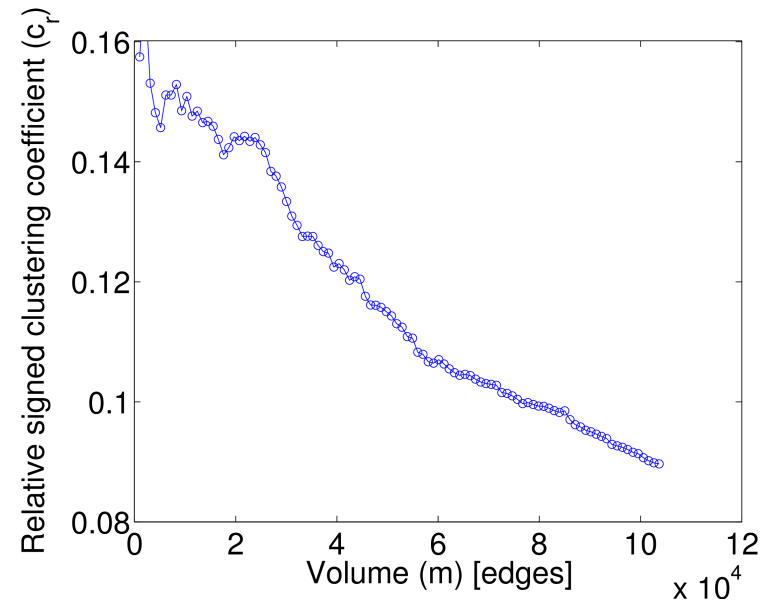
# Temporal Analysis of $C_2$



# Temporal Analysis of $C_3$

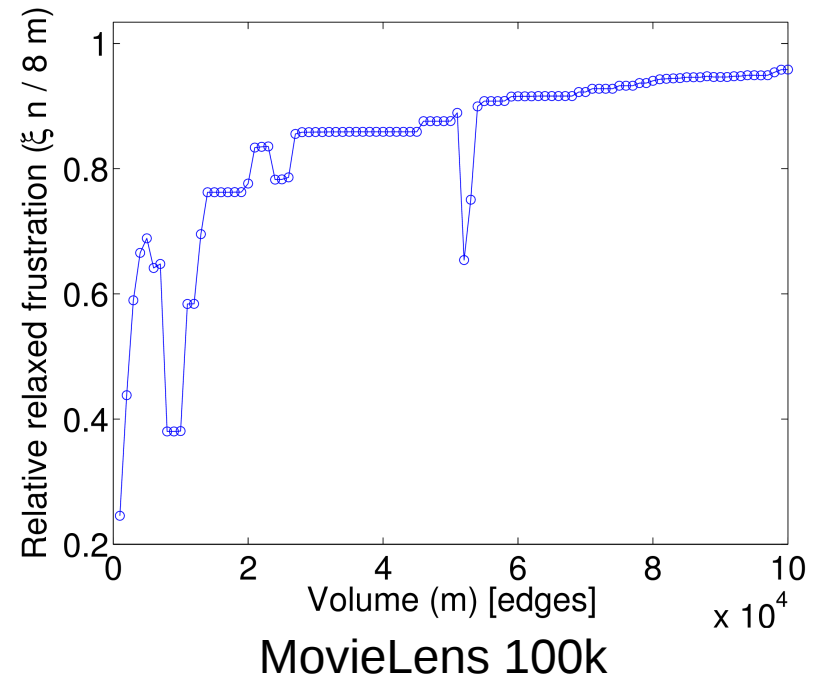
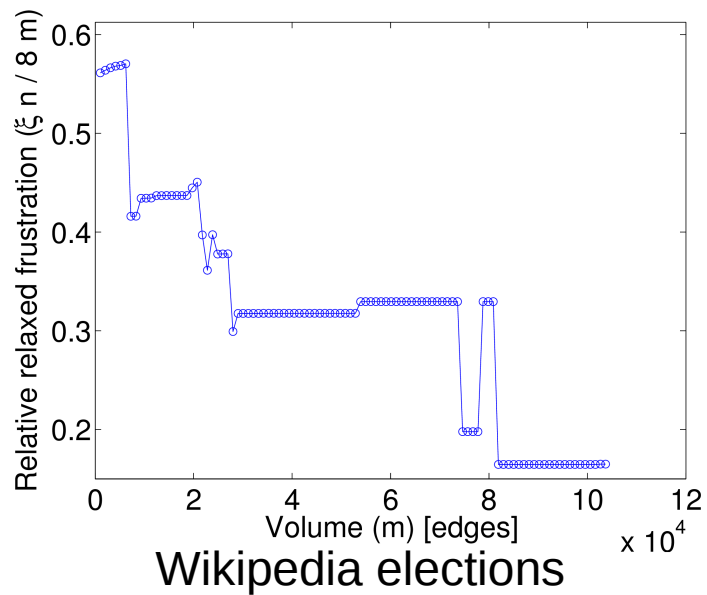


Epinions ratings

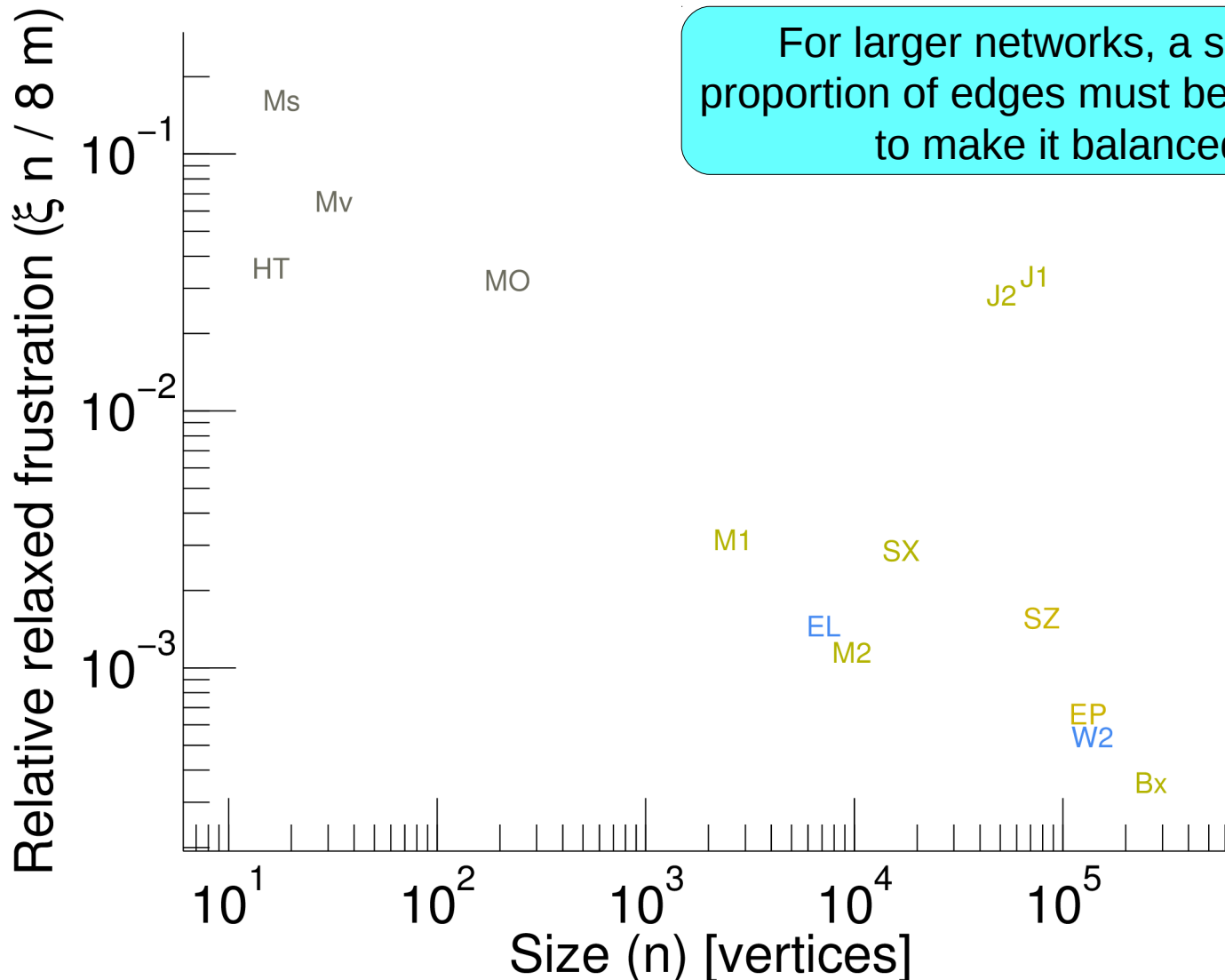


Wikipedia elections

# F\* over Time



# Cross-Dataset Comparison of $F^*$





# Merci

[konekt.uni-koblenz.de](http://konekt.uni-koblenz.de)

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We want more datasets with negative edges, and timestamps.

- In particular: with changing and/or disappearing edges